Power Analysis of a Single Degree of Freedom (DOF) Vibration Energy Harvesting System Considering Controlled Linear Electric Machines

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Abstract— In this paper, the power analysis of a single degree of freedom (DOF) is presented considering the controlled linear electric machines. The vibration energy harvesting system is modeled by tuned mass-damper models, subject to a sinusoidal force. Then the linear electrical machine is controlled to provide the extra damping force to vibration energy systems, and the damping force is modeled in terms of the electric stiffness and the electric damping coefficient. Therefore, in the vibration energy harvesting systems, linear electric machines have two functions: vibration control and energy harvesting. The generated electric power is analyzed for a single DOF energy harvesting system in different operating conditions. The power analysis of the vibration energy harvesting systems shows that the generated electric power is greatly impacted by the dimensionless excitation frequency, the electric stiffness, and the electric damping coefficient/ratio. To improve both the electric power generation and vibration control, the required electric stiffness, and the electric damping coefficient/ratio provided by the linear electric machines must be selected.

I. INTRODUCTION

In response to the demand for clean, highly efficient energy sources, energy harvesting has emerged as one of the most promising technologies in recent years. Vibration has been one of the most widespread sources of energy; the vibration energy from human motions, building/bridge vibration, railway vibration, and vehicle systems can produce electric power ranging from tens of microwatts up to several megawatts [1-5]. The models of vibration energy systems are presented based on a single degree of freedom (DOF) spring-mass-damper model or a multi-DOF spring-mass-damper model [6-7]; the power analysis of single DOF and multi-DOF vibration energy harvesting systems are provided in [6-7].

However, in most previous published models of vibration energy harvesting systems, the phase current is not controlled in the electromagnetic energy harvesters (linear electric machines) [6] as shown in Fig. 1. The electromagnetic energy harvester is connected to the external circuit, including a resistor and an inductor. The equations of the model can be expressed by (1-3). However, these parameters of the external circuit, including the load resistor and the inductor, are assumed constant in order to simplify the power analysis. In this case, the phase current is not controlled and will be directly determined by Equation (3), and thus the extra damping force provided by the linear electric machine defined by Equation (2) is not controlled.

\[ m\ddot{x} + k_s x + c_s \dot{x} + f_e = F_1 \sin(\omega_t t) \]  \( (1) \)

\[ f_e = k_{fe} \]  \( (2) \)

\[ e_1(x_1, i_1) = k_{emf} \dot{x}_1 = R_1i_1 + \frac{di_1}{dt} \]  \( (3) \)

where \( x_1 \) is the displacement of the mass \( m_1; F_1 \) and \( \omega_t \) are the magnitude and the angular frequency of the sinusoidal force, respectively; \( k_s \) and \( c_s \) are the mechanical stiffness and the mechanical damping coefficient of the mass \( m_1 \), respectively; \( k_{fe} \) and \( k_{emf} \) are the coefficients of the force and the back EMF provided by the certain type of linear electric machines, respectively; \( R_1, L_1, e_1, \) and \( i_1 \) are the load resistance, the inductance, the back EMF, and the phase current, respectively.
Therefore, the close-loop control of the linear electric machines is eliminated in vibration energy harvesting systems; benefits of using the linear electric machines are undermined. Linear electric machines are playing a key role in vibration energy harvesting systems: controlling vibration and harvesting energy. To improve both the electric power generation and vibration control, the damping force provided by the linear electric machines must be controlled.

In this paper, the power analysis of a single degree of freedom (DOF) vibration energy harvesting system is presented considering the controlled damping force provided by linear electric machines. The controlled damping force is modeled in terms of the electric stiffness and the electric damping coefficient. A comprehensive analysis of the factors influencing the power generation and vibration control of the vibration energy harvesting systems will be provided. Based on the analysis, the possible solutions to improve the electric power generation and vibration control of vibration energy harvesting systems will be discussed by the end.

II. POWER ANALYSIS OF A SINGLE DOF VIBRATION ENERGY HARVESTING SYSTEM

The mechanical behavior of a single DOF vibration energy harvesting system shown in Fig. 2 is expressed by Equation (4) when subject to the sinusoidal force excitation. The damping force $f_d$ provided by linear electric machines is a function of the phase current, which can be controlled as Equation (5) by controlling the phase current in the equivalent circuit model of the linear electric machine in Equation (6). The phase voltage in Equation (6) can be determined by the corresponding force control strategies through power electronics converters. The back EMF is a function of phase current and velocity. Depending on the type of linear electric machines, the functions of the damping force and/or the back EMF may vary. For instance, in the linear DC electric machine, the damping force and back EMF are linear functions of the phase current and velocity, respectively.

$$m_1\ddot{x}_1 + k_1x_1 + c_{1e}\dot{x}_1 + f_{1e} = F_1\sin(\omega t)$$  \hspace{1cm} (4)

$$f_{1e} = f_i(k_1) = k_{ue}x_1 + c_{ue}\dot{x}_1$$  \hspace{1cm} (5)

$$e_i(x_1, i) = R_i i + L_i \frac{di}{dt} + u_i$$  \hspace{1cm} (6)

where $x_1$ is the displacement of the mass $m_1$; $F_1$ and $\omega$ are the magnitude and angular frequency of sinusoidal external force; $k_1$ and $c_{ue}$ are the mechanical stiffness and the mechanical damping coefficient of the mass $m_1$, respectively; $k_{ue}$ and $c_{ue}$ are the controlled electric stiffness and the electric damping coefficient provided by the electric damping force $f_{1e}$, respectively; $R_i$, $L_i$, $e_i$, $u_i$, and $i$ are the ohmic resistance, the inductance, the back EMF, the phase voltage, and the phase current in the equivalent circuit model of linear electric machines, respectively.

It should be noted that in previous published work, the linear electric machine is directly connected to the constant load, and thus the controllable phase voltage in their equivalent circuit models are neglected. Due to the uncontrolled phase current, the electric damping force is not controlled. For this reason, the damping force may not be expressed as an explicit function of the displacement and the velocity. Only when the pure resistor is connected to the linear electric machine, Equation (3) is reduced to

$$e_i(x_1, i) = k_{ue}x_1 = R_i i$$  \hspace{1cm} (7)

Then the electric damping force in Equation (2) can be represented by a linear function of the velocity in (8).

$$f_{1e} = k_{ue} = \left(\frac{k_e}{R_i}\right)\dot{x}_1$$  \hspace{1cm} (8)

Compared to the electric damping force in Equation (5) for a single DOF vibration energy harvesting system, the electric damping coefficient in this case is a constant and the electric stiffness is set to zero.

Unlike the previously published work, in this paper, the electric damping stiffness $k_{ue}$ and the electric damping coefficient $c_{ue}$ can be adjusted instantaneously by adjusting the electric damping force, which can be controlled by the phase current of the linear electric machines. This will leverage the potential of the linear electric machines in terms of the electric power generation and vibration control in vibration energy harvesting systems.

![Fig. 2. Illustration of a single-DOF energy harvesting system.](image)

By combing (4) and (5), the model of the energy harvesting systems can be represented as Equation (9).

$$m_1\ddot{x}_1 + k_1x_1 + c_1\dot{x}_1 = F_1\sin(\omega t)$$  \hspace{1cm} (9)

where

$$c_1 = c_{ue} + c_{ue}$$

$$k_1 = k_{ue} + k_{ue}$$

$159$
When the system reaches the steady state, \( x_1 \) is represented by (10).

\[
 x_1 = X_1 \sin(\omega t - \theta) \tag{10}
\]

where

\[
 X_1 = \frac{F}{\sqrt{(k_1 - \alpha_1 m_1)^2 + c_1^2 \alpha_1^2}}
\]

\[
 \theta = \arctan\left(\frac{c_1 \alpha_1}{k_1 - m_1 \alpha_1}\right)
\]

The average electric power \( P_{ave} \) per cycle is given by (11).

\[
P_{ave} = \frac{1}{2} c_1 e_1 \frac{\dot{x}_e \ddot{x}_e}{F} = \frac{1}{2} c_1 e_1 \alpha_1^2 = \frac{1}{2} e_1 c_1^2 F_e^2 \tag{11}
\]

\[
= \frac{1}{2} c_1 e_1 \frac{F_e^2}{(k_1 + k_{te} - m_1 \alpha_1)^2 + (c_1 + c_{te})^2}
\]

In this paper, \( k_{te} \) and \( c_{te} \) can be adjusted in order to improve the average electric power generated by a single DOF vibration energy harvesting system. The optimal \( k_{te} \) and \( c_{te} \) are firstly selected to maximize electric power generation. To obtain the optimal damping coefficient \( c_{te} \), the derivative of \( P_{ave} \) over \( c_{te} \) is set to zero as shown in Equation (12).

\[
\frac{\partial P_{ave}}{\partial c_{te}} = 0 \tag{12}
\]

By solving (12), the optimal \( c_{te} \) can be obtained as (13). It should be noted that the optimal \( c_{te} \) is a function of the electric stiffness \( k_{te} \), which needs to be determined.

\[
c_{te}^{opt} = \sqrt{c_1^2 + \left(\frac{k_1 + k_{te} - m_1 \alpha_1}{\alpha_1}\right)^2} \tag{13}
\]

To obtain the optimal damping stiffness \( k_{te}^{opt} \), the derivative of \( P_{ave} \) over \( k_{te} \) is set to zero as shown in Equation (14).

\[
\frac{\partial P_{ave}}{\partial k_{te}} = 0 \tag{14}
\]

By solving (14), the optimal \( k_{te} \) can be obtained as (15).

\[
k_{te}^{opt} = m_1 \alpha_1^2 - k_{te} \tag{15}
\]

After \( k_{te} \) is selected as the optimal value, the optimal \( c_{te} \) must be the same as \( c_{te} \) in order to maximize the electric power generation based on Equation (13). The maximum electric power in this case can be obtained as (16).

\[
P_{ave, k_{te}^{opt}, c_{te}^{opt}} = \frac{1}{2} c_1 e_1 \frac{F_e^2}{8 c_{te}^{opt}} \tag{16}
\]

If \( k_{te} \) is set to zero as suggested in the most energy harvesting systems, the average electric power \( P_{ave} \) per cycle can be expressed by (7).
\[ \bar{x}_{i, \text{Normalized}} = \frac{x_i}{x_0} = \frac{k_{1i}}{\sqrt{(k_i - \alpha_i^2)^2 + c_i \alpha_i^2}} \]  
(24)

\[ \bar{x}_{j, \text{Normalized}} = \frac{1}{\sqrt{(1 - \alpha_i^2)^2 + [2\alpha(\zeta_{1i} + \zeta_{1s})]}} \]  
(25)

The power output under the static force \( F_1 \) is defined by (26).
\[ P_1 = \frac{F_1^2}{\omega_1 m_1} \]  
(26)

The dimensionless power can be obtained by (27). Only when Equation (19) is valid, (28) can be obtained.
\[ P_{\text{ave, Normalized}} = \frac{\frac{1}{2} c_{1e} \alpha_i^2 \omega_1 m_1}{k_1 - m_1 \alpha_i^2 + c_i \alpha_i^2} \]  
(27)

According to Equation (28), \( \alpha \) and \( \zeta_{1e} \) can be adjusted in order to improve the average power generated by the vibration energy harvesting system.

By solving (29), the optimal \( \zeta^{\text{opt}}_{1e} \) can be obtained as (30) to achieve the maximum average power generation.
\[ \frac{\partial P_{\text{ave, Normalized}}}{\partial \zeta_{1e}} = 0 \]  
(29)

\[ \zeta_{1e}^{\text{opt}} = \sqrt[4]{\frac{(1 - \alpha_i^2)^2}{4\alpha^2} + \zeta_{1s}^2} \]  
(30)

By substituting (30) into (28), the maximum dimensionless power is given by (31).
\[ \bar{P}_{\text{max, Normalized}} = \frac{\alpha}{4\sqrt{(1 - \alpha_i^2)^2 + 4\alpha^2 \zeta_{1s}^2 + 8\alpha \zeta_{1s}}} \]  
(31)

For solving Equation (32), the optimal \( \alpha \) can be obtained by (33) and then the dimensionless maximum power is shown as Equation (34).
\[ \frac{\partial P_{\text{ave, Normalized}}}{\partial \alpha} = 0 \]  
(32)

\[ \alpha^{\text{opt}} = 1 \]  
(33)

\[ \bar{P}_{\text{max, Normalized}} = \frac{1}{16\zeta_{1s}} \]  
(34)

IV. SIMULATION VALIDATION OF A SINGLE DOF VIBRATION ENERGY HARVESTING SYSTEM

In this section, a simple example of a single DOF energy harvesting system is provided to demonstrate the impact of the dimensionless excitation frequency, the electric stiffness, and the electric damping ratio/coefficient on the generated electric power and vibration control. Two cases, including the non-dimensionless analysis and the dimension analysis, will be investigated. As previously discussed, the dimensionless analysis can only be performed when the electric stiffness is set to zero. Therefore, the non-dimensionless analysis is firstly conducted to investigate the impact of the electric stiffness and electric damping coefficient on the electric power generation and vibration control. Then the dimensionless analysis is included to demonstrate the impact of the dimensionless excitation frequency and the electric damping ratio on the generated electric power and vibration control.

1) Non-dimensionless Analysis

The parameters of the single degree of freedom (dof) spring-mass-damper system are: \( m_1 = 10; k_1e = 1; c_{1e} = 1\%; \omega_1 = 0.3126; F_1 = 1 \). Fig. 3 shows the average power in terms of \( \omega / \omega_1 \) and \( c_{1e} \) when \( k_{1e} = 0 \). As shown in Fig. 3, the generated electric power reduces very quickly when the excitation frequency is far from the undamped natural frequency. Fig. 4 shows the maximum electric power when the optimal \( c_{1e} \) is selected and \( k_{1e} \) is set to zero. Fig. 5 shows the ratio \( r \) which is defined by (35). As shown in Fig. 5, the optimal electric stiffness \( k_{1e} \) can substantially increase the average electric power compared to the most conventional energy harvesting system where the electric stiffness \( k_{1e} \) is set to zero. Therefore, by adjusting the electric stiffness, the electric power generated by a single DOF vibration energy harvesting system can be greatly enhanced, which further demonstrate the benefits of the closed-loop force control of the linear electric machines.

\[ r = \frac{P_{\text{ave, Normalized}}}{P_{\text{ave, Normalized}} \mid k_{1e} = 0} \]  
(35)

Fig. 3. The average power in terms of \( \omega / \omega_1 \) and \( c_{1e} \) when \( k_{1e} = 0 \).
Fig. 4. The maximum electric power when the optimal $c_{1e}$ is selected and $k_{1e}$ is set to zero.

Fig. 5. Ratio $r$ which is defined by Equation (36).

2) Dimensionless Analysis

For the dimensionless analysis, $\zeta_{1m}$ is set as 2%. In this section, the dimensionless power analysis and the dimensionless displacement analysis of a single DOF vibration energy harvesting system are performed in various normalized excitation frequencies and electric damping ratios.

Fig. 6 shows the dimensionless average power in terms of $\alpha$ and $\zeta_{1e}$. Fig. 7 shows the dimensionless displacement in terms of $\alpha$ and $\zeta_{1e}$. As shown in Figs. 6 and 7, when the normalized excitation frequency is one, the dimensionless electric power and displacement reaches its peak value. For the same $\alpha$, both the dimensionless average power and the dimensionless displacement vary significantly with $\zeta_{1e}$. When $\zeta_{1e}$ is equal to its optimal value, the maximum power occurs; the displacement is medium. Therefore, in order to improve both electric power generation and vibration control, the required $\zeta_{1e}$ needs to be selected. When the normalized excitation frequency is one, the optimal $\zeta_{1e}$ is equal to $\zeta_{1m}$.

Fig. 8 shows the dimensionless average power when $\zeta_{1e}=\zeta_{1m}$ and $\zeta_{1e}=\zeta_{1e}^{opt}$, respectively. Fig. 9 shows the dimensionless displacement when $\zeta_{1e}=\zeta_{1m}$ and $\zeta_{1e}=\zeta_{1e}^{opt}$, respectively. Compared to the case ($\zeta_{1e}=\zeta_{1m}$), the case ($\zeta_{1e}=\zeta_{1e}^{opt}$) generates higher average power, but lower displacement. Therefore, the case ($\zeta_{1e}=\zeta_{1e}^{opt}$) is a better candidate in terms of electric power generation and vibration control.
V. CONCLUSIONS

In this paper, the power analysis of a single DOF vibration energy harvesting system has been presented considering the controlled linear electric machines. The power analysis of the vibration energy harvesting system shows that the generated electric power is greatly impacted by dimensionless excitation frequency, the electric damping coefficient/ratio, and electric stiffness provided by the linear electric machines. The maximum electric power occurs when the excitation frequency equals to the undamped natural frequency; beyond the undamped natural frequency, the generated electric power reduces very quickly. To improve the electric power generation, the optimal electric stiffness and the electric damping coefficient/ratio must be selected and then adjusted. In addition to the optimal electric damping coefficient/ratio, the adjustable electric stiffness can substantially increase the average electric power generation.

REFERENCES