Position Sensorless Control of Switched Reluctance Motor Based on Numerical Method

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Abstract—In this paper, a new position sensorless control method for switched reluctance motor drives is proposed. Rotor position is initially calculated based on the flux linkage-position-phase current characteristics by numerical method. Then, a third-order phase locked loop considering the acceleration variation is designed to undermine the impact of current sampling noise and numerical residual error on the estimated rotor position. Simulation and experimental results show that the proposed position sensorless control method has achieved sufficient accuracy in terms of position and speed estimation.

Index Terms—Position sensorless control, Numerical method, Phase locked loop, Switched reluctance motor (SRM).

I. INTRODUCTION

Switched reluctance motor (SRM) is an attractive candidate to variable speed drive applications. Compared to the widely used brushless DC (BLDC) machine and induction machine (IM) drive systems, SRM has features such as simple, robust and low-cost structure, high reliability and satisfactory high speed performance [1].

A typical SRM drive system is shown in Fig. 1. Normally an SRM is driven by asymmetric half bridges. Current controller is employed to generate switching signals for the asymmetric half bridges according to the current reference and rotor position. The current reference is either given by a speed controller or a torque distributor. If the current reference comes directly from a speed controller, flat top chopping current for each phase is employed. Due to the strong nonlinearity, in some cases, the flat top chopping current regulation might not provide satisfactory performance. Therefore, torque sharing control is used to produce constant torque [2]–[7].

According to Fig. 1, both the current controller and the torque distributor need the rotor position information. The rotor position is often obtained through external sensors such as encoder, resolver, hall sensors or optical couplers. These sensors increase the cost of the SRM system and reduce the system reliability. In order to compete with the widely used position sensorless controlled BLDC and IM drive system in terms of cost and reliability, the sensorless control method for SRM drive has to be developed.

There are two categories of position sensorless control methods for SRMs: flux-linkage based methods and inductance based methods. Since phase inductance is a function of rotor position, rotor position is detected through injecting current pulses to the inactive phase in [8], [9]. Commutation happens when the injected current peak exceeds a certain threshold. However, current injection methods are limited at higher speed. In [10]–[14] the phase inductance is measured through the current and voltage on the active phase, which relies on the instant measurement of current derivatives. Therefore, this method suffers from the measurement noise. In [15]–[21], the rotor position is detected based on flux linkage-current-position characteristics. The current observer or flux observer or hybrid observer can be designed to estimate the rotor position indirectly. In [22], [23] instead of comparing the estimated rotor position with the commutation angles, the estimated flux is compared with the commutation flux. The flux or current observer methods work well at constant speed, but will have significant error during acceleration and deceleration. In [24], [25], rotor position is calculated directly based on flux linkage-position-current characteristics with either lookup table or numerical method, which also suffers from measurement noise.

In this paper, a new position sensorless control method for SRM is proposed. The new method is a combination of direct calculation method and flux observer method. Rotor position is firstly calculated by numerical method through the relationship among position, phase current and flux linkage. Then the motion of the SRM including acceleration is modeled and a phase locked loop (PLL) based on this model is designed to estimate the rotor according to the numerically calculated results. The dynamic response of this method is improved through the motion model and the impact of measurement noise and residual error could be reduced through parameter selection of the PLL. Both simulation and experimental results are provided to verify the performance of the proposed position sensorless control method.
II. MODEL OF SRM

By neglecting mutual coupling between phases, the phase voltage equation of SRM is given as:

\[ u_w = R_w \cdot i + \frac{d\psi(\theta, i)}{dt} \]  

(1)

where \( u_w \) is the phase voltage applied on the phase winding; \( R_w \) is the winding resistance; \( \psi \) is the flux linkage; \( \theta \) is the rotor position and \( i \) is the phase current. Due to its double salient structure and saturation, \( \psi \) is a nonlinear function of both \( i \) and \( \theta \). Fig. 2 shows the flux linkage profile of the studied SRM in this paper. \( i \) could be measured directly, but \( \theta \) is not available without position sensor. In order to obtain the rotor position, \( \psi \) has to be measured by integrating (1):

\[ \psi = \int (u_w - R_i \cdot i) \, dt \]  

(2)

With the flux linkage profile shown in Fig. 2, \( \psi \), and \( i \) obtained, \( \theta \) could be calculated either directly by lookup table or by numerical method. In this paper, numerical method is adopted to calculate the rotor position.

Since \( \psi(\theta, i) \) is a function of \( \theta \) and \( i \), in a small neighborhood of a point in Fig. 2, there is

\[ \Delta \psi = \frac{\partial \psi}{\partial i} \bigg|_{\theta=\text{const}} \cdot \Delta i + \frac{\partial \psi}{\partial \theta} \bigg|_{i=\text{const}} \cdot \Delta \theta \]  

(3)

Since current is directly measured, \( \Delta i \) could be taken as zero. Then, (3) could be written as

\[ \theta - \hat{\theta} = \frac{\partial \theta}{\partial \psi} \bigg|_{i=\text{const}} \cdot (\psi - \hat{\psi}) \]  

(4)

where \( \hat{\theta} \) is the estimated rotor position near the real position and \( \hat{\psi} \) is the corresponding estimated flux linkage.

The motion equations of the SRM is:

\[ \dot{\theta} = \omega \]

\[ \dot{\omega} = \alpha \]  

(5)

where \( \omega \) is the angular speed, \( \alpha \) is the angular acceleration. \( \alpha \) is determined by factors such as torque production, load condition, load inertia, and friction.

III. PROPOSED SENSORLESS CONTROL METHOD

A. Position estimation

Since the controller is digitally implemented, the estimated position for current sampling step comes from the previous sampling step and is donated as \( \hat{\theta}(k|k-1) \). Then the corresponding estimated flux is obtained as

\[ \hat{\psi}(k|k-1) = \psi(\hat{\theta}(k|k-1), i(k)) \]  

(6)

According to (4), there is

\[ e(k) = \hat{\theta}(k|k) - \hat{\theta}(k|k-1) \]

\[ = \frac{\partial \theta}{\partial \psi} \bigg|_{i(k)} \cdot (\psi(k) - \hat{\psi}(k|k-1)) \]  

(7)

where \( \hat{\theta}(k|k) \) is the estimated rotor position of current step and \( e(k) \) is the estimation error.

\[ g = \frac{\partial \theta}{\partial \psi} \] is the iteration gain of the numerical calculation, whose values at different currents are shown in Fig.3(a). It is shown that \( g \) varies a lot with rotor position. Normally, the values are stored in a lookup table for use. In order to save time and space of the digital controller, only the smallest value at each current is taken as shown in Fig.3(b). This approach will not lead to large errors considering relatively flat curve when the phase is turned on. Smaller gain may lead to slower convergence, which will not affect the stability of the numerical iteration.
B. PLL design

In order to reduce the impact of measurement noise and numerical residual error, a PLL is designed according to (5):

\[
\hat{\theta} = \omega + k_0 e \\
\dot{\omega} = \dot{\alpha} + k_\omega e \\
\dot{\alpha} = k_\alpha e
\]  

(8)

where \(\hat{\theta}\), \(\dot{\omega}\) and \(\dot{\alpha}\) are the estimated position, angular speed and angular acceleration respectively. \(e\) is the estimation error.

According to (8) and (5), the transfer function of the error dynamics of the system becomes

\[
e \over \dot{\alpha} = \frac{1}{s^3 + k_0 s^2 + k_\omega s + k_\alpha}
\]

(9)

(9) is a third order system and is stable as long as the gains of the PLL are positive.

Since the controller is implemented in digital processor, (8) has to be digitalized:

\[
\hat{\theta} (k+1|k) = \hat{\theta} (k|k-1) + \dot{\theta} (k|k-1) \cdot T + k_0 \cdot e (k) \cdot T \\
\dot{\omega} (k+1|k) = \dot{\omega} (k|k-1) + \dot{\alpha} (k|k-1) \cdot T + k_\omega \cdot e (k) \cdot T \\
\dot{\alpha} (k+1|k) = \dot{\alpha} (k|k-1) + k_\alpha \cdot e (k) \cdot T
\]

(10)

Combing (10) and (7), the proposed position estimation is obtained.

C. Relationship Between Previous Sensorless Control Methods

The proposed position sensorless control method is similar to some previously proposed sensorless control method. For example, if the \(\frac{\partial \hat{\theta}}{\partial \psi}\) item is removed from (7), and the PLL is reduced to second order, then the position estimator becomes:

\[
\hat{\psi} (k|k-1) = \hat{\psi} (\hat{\theta} (k|k-1), i (k)) \\
e (k) = \left( \psi (k) - \hat{\psi} (\hat{\theta} (k|k-1) \right)
\]

\[
\hat{\theta} (k+1|k) = \hat{\theta} (k|k-1) + \dot{\theta} (k|k-1) \cdot T + k_0 \cdot e (k) \cdot T \\
\dot{\omega} (k+1|k) = \dot{\omega} (k|k-1) + \dot{\alpha} (k|k-1) \cdot T + k_\omega \cdot e (k) \cdot T
\]

(11)

which is a typical flux observer based position estimator.

If the third order PLL is removed, then (7) becomes

\[
\hat{\theta} (k) = \hat{\theta} (k-1) + \frac{\partial \theta}{\partial \psi} \bigg|_{i(k)} \cdot \left( \psi (k) - \hat{\psi} (k-1) \right)
\]

(12)

which is a numerical method based direct calculation method.

Therefore, the proposed method could be considered as the combination of observer based method and direct calculation method.

| TABLE I. Calculating the estimated positions at the end of start up stage. |
|--------------------------|--------------------------|
| **If** | **Then** |
| \(\theta_a > \theta_b, \theta_c > \theta_b, \theta_b > \theta_a\) | \(\theta_a = 360 - \theta_a, \theta_c = 360 - \theta_c\) |
| \(\theta_a < \theta_b, \theta_c > \theta_b, \theta_b > \theta_a\) | \(\theta_a = 360 - \theta_a, \theta_c = 360 - \theta_c\) |
| \(\theta_a < \theta_b, \theta_c < \theta_b, \theta_b < \theta_a\) | \(\theta_a = 360 - \theta_a, \theta_c = 360 - \theta_c\) |
| \(\theta_a > \theta_b, \theta_c < \theta_b, \theta_b < \theta_a\) | \(\theta_a = 360 - \theta_a, \theta_c = 360 - \theta_c\) |
| \(\theta_a > \theta_b, \theta_a < \theta_c, \theta_b < \theta_a\) | \(\theta_a = 360 - \theta_a, \theta_c = 360 - \theta_c\) |

IV. DIGITAL IMPLEMENTATION

A. Start Up

Since there is no information about the rotor position before start up, a start up sequence is required. On start up, current pulses are injected to all three phases of the SRM for a short period of time. The position estimators of the three phases start to estimate rotor positions at the same time. Since Fig. 2 is symmetric between (0,180] and [180,360], the three estimators should only estimate the rotor position among (180,360] in this stage. After the estimated positions converges and becomes stable, the rotor position is calculated through the estimated values according to TABLE I.

There is known relationship among the three phase positions, which is:

\[
\theta_a = \theta_b + 120 \\
\theta_b = \theta_c + 120
\]

Therefore, a more accurate rotor position could be obtained by averaging the three calculated positions:

\[
\hat{\theta} = \frac{\theta_a + \theta_b + \theta_c + 240}{3}
\]

(14)

With \(\hat{\theta}\) obtained, the SRM could be started by keeping turning on the active phase and turning off the inactive phases.

B. Low Speed

Since the flux linkage is calculated by (2) by integration in an open loop manner, it will lose its accuracy when speed is low and the integration time is long. In this case, current pulses are injected into the inactive phases to help estimating the rotor position and the position estimator of the active phase is disabled.

C. High Speed

After the SRM gets into higher speed, the calculated flux linkage is accurate enough. In this stage, the pulse injection is stopped. Rotor position is estimated by the active phase.

V. SIMULATION RESULTS

Simulation is performed in MATLAB/SIMULINK to verify the effectiveness of the proposed position sensorless control method. Since speed control and torque control is not the main concern of this paper, only current control is applied. PI current controller is adopted at start up and low speed, while proportional flux controller with feed-forward is adopted at high speed. The controller enters high speed mode when the speed is higher than 410 RPM. Considering the sampling noise
in practice, the control diagram of the system is shown in Fig. 4. Speed is controlled by varying the load torque and the speed is control to ramp from zero to 4,500 RPM within 1 second.

The parameters of the system as well as the proposed PLL are shown in TABLE II. The roots of (9) are -887.4438, -111.5460 and -1.0102. They are all located on the negative real axis which means the response of error does not have overshoot. The bode diagram of (9) is shown in Fig. 5 with the parameters shown in TABLE II. It is shown from Fig. 5 that the error is significantly attenuated.

A. Start Up and Low Speed

Fig.6 shows the three phase currents and estimated rotor positions during start up and low speed. The reference current of 10 A is injected to all three phases. Then the calculation in TABLE I is performed and the controller enters low speed region. It is shown that the start up period is very short (about 1ms), which doesn’t have much impact on the torque production of the SRM.

Since speed is low at this time, the model based flux calculation may suffer from integration errors. Fig.7 shows the flux linkage estimation error at low speed. Estimating rotor position with the active phase may cause significant error. Therefore, current pulses of about 5 A is injected to the inactive phases to estimated the rotor position.

The estimated rotor position of one phase and the real rotor
position are shown in Fig. 8. It is shown that the proposed estimation method has significant accuracy during start up and low speed operation.

### B. High Speed

The SRM is driven to 4,500 RPM (36,000 RPM in electric speed) to test its performance at high speed. Fig. 9 shows the estimated and real positions during high speed operation. Fig. 10 shows the real speed and estimated speed from the beginning. It is indicated that both the position estimation and speed estimation are accurate.

### C. Current Noise

In practice, the current sampling of ADCs may introduce noises to the sampled currents. To simulate this case, white noise is introduced to test the performance of the proposed sensorless control method. Fig. 11 shows the estimated rotor position with the sampled current and the real rotor position at 4,500 RPM in this case. It is shown that the estimated position matches the measured position very well and is merely effected by the noise. This is due to the noise attenuation ability of the third order PLL. Fig. 12 shows the position estimation error from the beginning. As a comparison, Fig. 13 shows the position estimation error without the third-order PLL. It is shown that, compared to Fig. 12, the estimation error increases significantly without the PLL.

![Fig. 8. Real and calculated rotor position during start up and low speed.](image8)

![Fig. 9. Three phase currents, estimated and real positions at 4,500 RPM.](image9)

![Fig. 10. Real and estimated rotor speed from start up.](image10)

![Fig. 11. The estimated and real positions with current noise at 4,500 RPM.](image11)

![Fig. 12. The position estimation error with the PLL.](image12)

![Fig. 13. The position estimation error without the PLL.](image13)

### VI. Experimental Results

Experiments are designed to verify the effectiveness of the proposed position sensorless control method. The experiments are performed on a 12/8 SRM. It’s flux linkage profile is shown in Fig. 2. A BLDC with diode rectifier is used as its load. Fig. 14(a) shows the tested SRM and the load BLDC. A DC/DC converter is connected to the output of the diode rectifier to adjust the load. Fig. 14(b) shows the power converter for the SRM and the DC/DC converter and the load resistor. A position sensor is installed on the tested SRM to measure the real position and speed. But this position and speed are only used for comparison purpose, and is not used by the proposed controller. The measured and estimated positions and speeds are sent to a PC for record through USB communication at 10kHz.

Firstly, the rotor is fixed to test whether the proposed method has the ability to start the SRM with heavy load.
The currents of phase A and phase B are sampled and shown in Fig. 15(a). Fig. 15(b) shows the estimated and measured position obtained in the experiment. It is shown that during the start up period, short pulses are injected to estimate the initial position. The estimated initial position is very close to the real position. The start up period is very short and thus it doesn’t have obvious impact on torque production. Then the controller enters low speed operation and pulses are injected on the inactive phases to estimate the rotor position. In the experiment, the rotor moves a little bit. At first, phase A is turned on, current pulses are injected to phase B. Then, phase B is turned on. At this time, the position is estimated by injecting current to phase C. Then phase A is turned off and current pulses are injected to phase A. The estimated position is very close to the real position.

Secondly, the SRM is controlled to run at 1,000 RPM and 4,500 RPM respectively to test the effectiveness of the proposed position sensorless control method. Fig. 16 shows the real and estimated position at start up and low speed operation. Due to the large coggging torque of the load BLDC, the rotor starts spinning with vibration. This high dynamic introduces errors in the beginning. With the increment of the speed, the inertia of the load smooths out the vibration and estimated position converges to the real position shortly.

Fig. 17 shows the estimated and real position at 1,000 RPM. It is shown that the position estimation is very accurate. Fig. 18 shows the estimated and real position at 4,500 RPM. It is shown that even though there are few sampling point in each period, the estimated position is still accurate enough. Fig. 19 shows the real and estimated speed. Fig. 20 shows the corresponding position estimation error. It is shown that the estimation response is fast and accurate enough that the estimated speed matches the real speed very well. The estimation error is within ±18° and decreases to be in ±9° as the speed acceleration decreases.

VII. CONCLUSION

A new position sensorless control method for switched reluctance motor drives is proposed in this paper. Rotor position is initially calculated by a numerical method. Then, a third-order phase locked loop considering the acceleration variation is designed to undermine the impact of current sampling noise and numerical residual error on the estimated rotor position. The variation of speed acceleration is considered in designing the PLL to guarantee tracking accuracy during acceleration and deceleration. By applying the PLL, the impact of current sampling noise on the estimated rotor position is
Fig. 17. The estimated and real positions at 1,000 RPM.

Fig. 18. The estimated and real positions at 4,500 RPM.

Fig. 19. The estimated and real speed at 4,500 RPM.

Fig. 20. The estimated position error.

well attenuated. Simulation and experimental results show that the proposed sensorless control method has significant accuracy on both position and speed estimation.

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REFERENCES


