Optimal Calculation Method for Control of Switched Reluctance Motor

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Abstract—In this paper, the optimization problem for switched reluctance motor (SRM) is discussed and the convex optimization method is applied to calculate SRM control parameters instead of parameter sweeping. An optimal approach to calculate the maximum torque at each speed is presented. Also, a method to optimize control parameters is presented to minimize the copper loss for the given speed and torque. Simulation shows that the proposed optimal method is much faster than conventional parameter sweeping method with acceptable calculation errors.

I. INTRODUCTION

Switched reluctance motor (SRM) is an attractive alternative to the widely used induction machine and synchronous machine because of its simple structure, low cost, high reliability and high-speed performance. However, SRM suffers from high torque ripples and acoustic noise due to its special double salient structure, which greatly blocks its popularity.

Fig. 1 shows a basic structure of a 6/4 SRM. As shown in Fig. 1, both its stator and its rotor are salient. The double salient structure leads to high nonlinearity of its flux linkage profile and brings difficulty to its control strategy[1]–[3]. Unlike conventional rotating magnet field machines, it is not easy to find a transformation of coordinates to decouple its control parameters. For a SRM, current chopping regulation is the most commonly used control method in low and middle speed region. There are usually three parameters to be controlled, the turn-on angle, the turn-off angle and the current amplitude limit. In high speed region, the current chopping regulation turns to the single pulse regulation, and there are only two parameters to be controlled, which are the turn-on angle and turn-off angle.

Different combinations of the three parameters lead to different performances. Researches are focused either on how to find the combinations of these parameters in order to maximize the power at the given speed, or to maximize torque per ampere (TPA) or efficiency at the given speed and power. [4] presents a method to find out the combination of the turn on and turn off angles that produce maximum power at a certain speed with fixed current limit. [5] figures out that in single pulse operation, there could be different combinations of turn-on and turn-off angles for a given speed and torque command. Moreover, [5] also indicates that the copper loss varies significantly with different turn on and turn off angles while the iron loss doesn’t change very much. [6] presents an online search method of turn off angles and current limit in order to achieve the best efficiency with offline turn-on angle . [7] demonstrates that by experiments the turn-on angle mainly determines the output power while the turn-off angle mainly determines the efficiency. Then an online control method is presented to adjust both the turn on angle and turn off angle.

However, for the control method mentioned above, the online methods usually bring perturbation to the system, which is undesirable in any application. The offline methods usually use fixed-step parameter sweeping or fixed step univariate search technique. The parameter sweeping technique is very slow. The univariate search technique is faster, however, doesn’t guarantee a globally optimized result without careful study. This paper introduces a faster offline optimal calculation method to calculate the combinations of the control parameters.

II. ATTRIBUTES OF SRM OPTIMIZATION PROBLEM

Normally a SRM is driven by an asymmetric half bridge converter, as shown in Fig. 2. The inductors in Fig. 2 represent the phase windings of SRM. Since each bridge is asymmetric, the current in each phase winding is unidirectional. The relative position of stator and rotor has states during operation as shown in Fig. 3. To simplify the analysis, only one stator teeth and one rotor teeth are taken into consideration. When the rotor teeth is approaching the stator teeth, as shown in Fig. 3a, the stator winding should be energized so that a magnet force is generated to force the rotor move towards the stator. When the stator is aligned with the stator, as shown in Fig. 3b, even though there is force

Fig. 1 structure of a 6/4 SRM
between the stator and rotor, this force doesn’t contribute to the motion of the rotor. So at this time, the stator winding should be demagnetized so that there is no current in the winding and there is no force between stator and rotor. When the rotor teeth is leaving the stator teeth, as shown in Fig. 3c, the current in the stator winding is not desired since it will generate a magnet force to drag the rotor teeth back.

Fig. 5 shows the flux linkage curve and energy conversion loop in one stroke when operating a real SRM. It could be seen since the flux linkage profile shown in Fig. 4 is nonlinear, the flux linkage curve is also higher nonlinear.

The electromagnetic energy converted per stroke could be expressed as

\[ W = \int_{\theta(t)} i(t) d\psi(t) \]  \hspace{1cm} (1)

Where \( W \) is the electromagnetic energy converted by SRM per stroke, \( i(t) \) is the phase current and \( \psi(t) \) is the flux linkage. \( \theta(t) = \theta_0 + \omega t \) is the rotor position, \( \theta_0 \) is the initial position of the SRM, \( \omega \) is the angular speed. The phase current \( i(t) \) is a function of the flux linkage \( \psi(t) \) and rotor position \( \theta \).

\[ i(t) = f_i(\psi(t), \theta(t)) \]  \hspace{1cm} (2)

The flux linkage \( \psi(t) \) could be expressed as

\[ \psi(t) = \int_{0}^{t} [u(t) - Ri(t)] dt \]  \hspace{1cm} (3)

Where \( u(t) \) is the voltage applied on the winding, \( R \) is the phase resistance. When current chopping control is applied, \( u(t) \) could be expressed as

\[ u(t) = \begin{cases} 
E, & \theta_{on} \leq \theta_0 + \omega t \leq \theta_{off} \text{ and } i(t) < i_{lim} \\
-E, & \theta_{on} < \theta_0 + \omega t < \theta_{off} \text{ and } i(t) > i_{lim} \\
0, & \text{else}
\end{cases} \]  \hspace{1cm} (4)

where \( R \) is the winding resistor. \( \theta_{on} \) is the turn-on angle and \( \theta_{off} \) is the turn-off angle. \( i_{lim} \) is the current limit, \( E \) is the voltage of the DC power supply.

(2) is the relationship between current, flux linkage and rotor position, which could be obtained either by analytical method [4] or by a look up table. Due to nonlinearity of SRM, both (2) and (4) are strong nonlinear functions. Look up tables are employed for (2). After (2) is obtained, (1) could be obtained by numerical integration. The accuracy of the numerical integration depends on the selected time step \( dt \) and the accuracy of the look up table. In practice, the look up table is usually obtained by Finite Element Analysis (FEA) or by experimental measurement. As an engineering approach, the shape of (1) could be calculated and studied to find out whether optimization of the calculation speed is available.

A 12/8 SRM is studied and the flux linkage profile is calculated by FEA and is shown in Fig. 4. The DC source voltage is set to 24V, and each stator winding resistance is 0.3\( \Omega \). There are 90 electric degrees per electric period. Assuming that the aligned position is 0°, the unaligned position is 45°.
In order to study the relationship between the output power and the turn-on and turn-off angles, \( \theta_{on} \) and \( \theta_{off} \) are swept at a mechanical speed of 2000RPM (4000 RPM in electric speed) with step of 1°. The current limit \( i_{lim} \) is set to 20A. Since the parameter sweeping is very time consuming, it is not necessary to calculate all the combinations of these parameters. Positive torque is produced only when the rotor position is in the induction rising area (45°, 90°). \( \theta_{on} \) could be anywhere between (0°, 90°), while \( \theta_{off} \) should not be smaller than 45°, otherwise positive torque is not produced. What’s more, \( \theta_{on} \) should be smaller than \( \theta_{off} \). Since the SRM is analyzed in steady state, the flux linkage before and after each electric period should keep the same, otherwise this is not a stable state under the studied speed, and the result should be dropped. Therefore, constrains for \( \theta_{on} \) and \( \theta_{off} \) are obtained as

\[
0^\circ < \theta_{on} < 90^\circ \\
45^\circ < \theta_{off} < 90^\circ \\
\theta_{on} < \theta_{off}
\]  

(5) defines a feasible set for \( \theta_{on} \) and \( \theta_{off} \). The SRM optimal problem could be only studied within this feasible set.

The calculated shape of (1) in its feasible set is shown in Fig. 6. It is shown that, within its feasible set, (1) is actually a concave function. In this case, convex optimization could be applied to accelerate the calculation speed.

### III. OPTIMIZATION PROBLEMS OF SRM

#### A. Finding out maximum torque at each speed

One of the SRM optimization problems is to find out how much power or torque the machine could produce at a certain speed. This value is usually helpful during the machine design process. The designer usually needs to know the maximum power or torque a designed machine could produce at each speed to check whether the design meets the requirements. Normally this is done by sweeping the turn-on and turn-off angles in FEA tools, which is very time consuming.

It could be observed from Fig. 6 that there exists a unique point that could convert the maximum power in one stroke. This point also produces the maximum average torque because the average torque could be calculated as

\[
\bar{T}_q = \frac{nNW}{2\pi}
\]

(6)

Where \( n \) is the number of rotor poles and \( N \) is the number of stator phases.

Therefore, this optimization problem could be written as:

\[
\max W(\theta_{on}, \theta_{off}, i_{lim}) \\
\text{s.t.} \\
0^\circ < \theta_{on} < 90^\circ \\
45^\circ < \theta_{off} < 90^\circ \\
\theta_{on} < \theta_{off} \\
i_{lim} = i_{max}
\]

(7)

Where \( i_{max} \) is the rated current or maximum current allowed by the SRM phase windings. (7) is a convex optimization with linear unequal constrains, which could be solved efficiently. The initial point could be selected within its feasible set, but have to avoid its unstable area. When selecting the initial point, the \( \theta_{on} \) and \( \theta_{off} \) should satisfy

\[
\theta_{off} - \theta_{on} < 45^\circ
\]  

(8)
Fig. 7 demonstrates the optimization progress. The points calculated are marked by the beginning and ending of arrows. As shown in Fig. 7, given an initial point, the optimization progress searches the maximum power point by only calculating a few \( \theta_{on} \) and \( \theta_{off} \) combinations instead of sweeping all the parameters. This is much more efficient than parameter sweeping.

As a comparison, parameter sweeping shown in Fig. 6 gets a maximum power of 301.21 W, with \( \theta_{on} \) being 27° and \( \theta_{off} \) being 73°, while the optimization method shown in Fig. 7 gets a result of 302.04 W, with \( \theta_{on} \) being 27.77° and \( \theta_{off} \) being 72.75°. This indicates the optimization method is faster and more accurate than conventional parameter sweeping.

B. Finding out the operation point with minimum loss

If an SRM is required to work with its maximum power or torque, there is only one set of parameters to be chosen. But under some situations, the SRM is required to work below its maximum power. As is shown in Fig. 6, there could be a set of parameters that meet the output power requirement. Different combinations of parameters in the set could deliver the same desired torque \( \bar{T}_d \) with different power losses. Some combinations of parameters are more efficient than others. If all the parameters \( (\theta_{on}, \theta_{off} \text{ and } I_{lim}) \) are swept to pick up the most efficient parameter combination, it is very time consuming. One optimization problem is to find out the most efficient combination of parameters at a fast speed.

The power loss of SRM includes both iron loss and copper loss. But as stated in [5], the iron loss doesn’t vary much with different turn on and turn off angle combinations. Moreover, it is not easy to calculate iron loss. In this case, if copper loss is minimized, the total power loss is considered to be minimized. In this case, only copper loss is needed to be considered in this off line calculation. In one electric period, the copper loss in each phase could be expressed as

\[
W_{loss} = \int_{t_{on}}^{t_{off}} i(t)^2 R dt
\]  

Where \( t_{on} \) is the time when the phase winding is turned on and \( T \) is one electric period. Same as (1), (9) could also be calculated by numerical integration. The constrains for \( \theta_{on} \) and \( \theta_{off} \) are the same as (5), the current limit should not exceed the SRM maximum phase current. In this case, the optimization problem could be expressed as

\[
\begin{aligned}
\min & \quad W_{loss}(\theta_{on}, \theta_{off}, i_{lim}) \\
\text{s.t.} & \quad 0^\circ < \theta_{on} < 90^\circ \\
& \quad 45^\circ < \theta_{off} < 90^\circ \\
& \quad \theta_{on} < \theta_{off} \\
& \quad i_{lim} < i_{max} \\
W(\theta_{on}, \theta_{off}, i_{lim}) = & \frac{2\pi\bar{T}_d}{nN}
\end{aligned}
\]  

(10) is a convex optimization problem with nonlinear equality constrains. It could also be solved efficiently by convex optimization methods.

IV. CALCULATION AND EXPERIMENTAL RESULTS

Firstly, the maximum torques at each speed from 50 RPM to 2000 RPM with the step of 50 RPM are calculated for the studied SRM as shown in Fig. 8a. The numerical integration time step is \( 10^{-5} \)s. The corresponding turn-on and turn-off angles at each speed are obtained at the same time, which are functions of speed as shown in Fig. 8b. These functions could be used to either to make up a look up table or to be fitted into polynomial functions to control the SRM. It takes only about 3 to 5 iterations to get the optimized results at each speed, which is much faster than that of the conventional parameter sweeping. With these results obtained, a SRM could be controlled to run at its maximum power.

![Fig. 7 demonstration of optimization progress](image)

![Fig. 8 calculated results of maximum torque (a) and corresponding turn-on and turn-off angles (b).](image)
Experiment is designed to verify the effectiveness of the calculated results. The studied 12/8 SRM is connected with another machine. There is a fan load in the machine. So all the power delivered by the SRM is used to overcome the fraction and fan load, as shown in Fig. 9a. The Asymmetric half bridge power converter and the control board are shown in Fig. 9b.

When the turn-on angle and turn-off angle are fixed at 45° and 75° respectively, the SRM could only run to 1012 RPM, delivering an electric magnet power of 115W. While if the optimal turn-on and turn-off angle are used, the SRM could run up to 1559 RPM, delivering an electric magnet power of 243W. The calculated power with optimal parameters under 1559 RPM is 236W, which matches very well with the experimental result. The calculated and experimental currents when the SRM is running at 1559 RPM are shown in Fig. 10. It is shown that they match very well. The small mismatch is due to factors such the difference of the FEA obtained flux linkage profile and the real profile, the variation of the DC bus, the mutual flux, iron loss and etc. even there is some mismatches, the accuracy is high enough for engineering application.

![Machine with fan load](image1)
![Tested SRM](image2)

![Power converter](image3)
![Control board](image4)

**Fig. 9 experimental setup.** (a) tested SRM and its load, (b) control board and power converter

![calculated current](image5)
![experimental current](image6)

**Fig. 10 calculated current and the experimental current**

Secondly, the optimal parameters for the required torques at different speeds from 50 RPM to 2000 RPM are calculated with a step of 50RPM. At each speed, the case of 25%, 50%, and 75% of its maximum torque are calculated respectively. The obtained turn-on angles, turn-off angles and current limits are shown in Fig. 11. These data could also be fitted into polynomial functions as shown in Fig. 11 for real-time control.

In Fig. 11, the data marked by × and + are the calculated data, and the curves are their corresponding polynomial fit results. Since (10) has equal constrains, it takes about 15 iterations to get the optimal results for every speed and reference torque combination, but is still much faster than sweeping the three parameters. The maximum mismatch of equality constrains is 2.8%, which is acceptable as an engineering approach. With these data obtained, the SRM could be controlled at any torque and speed with best efficiency.

**Fig. 11 calculated and fitted results of 25%, 50%, 75% of the maximum torque at different speed.** (a) turn-on and turn-off angles, (b) current limit
V. CONCLUSION

In this paper, the optimization problem of SRM is addressed and then the optimal method to calculate the maximum torque at each speed is presented. In addition, the optimal method to calculate turn-on angle, turn-off angle and current limit is presented to minimize the copper loss for the certain speed and torque. Real case calculation shows that the proposed optimal method works much faster than conventional parameter swiping with tolerable errors.

VI. REFERENCES