Battery State-of-Power Peak Current Calculation and Verification Using an Asymmetric Parameter Equivalent Circuit Model

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Abstract—In this paper, a higher fidelity battery equivalent circuit model incorporating asymmetric parameter values is presented for use with battery state estimation (BSE) algorithm development; particular focus is given to state-of-power (SOP) or peak power availability reporting. A practical optimization-based method is presented for model parameterization fitting. Two novel model-based SOP algorithms are proposed to improve voltage-limit-based power output accuracy in larger time intervals. The first approach considers first-order extrapolation of resistor values and open-circuit voltage (OCV) based on the instantaneous equivalent circuit model parameters of the cell. The second proposed approach, which is referred to as multistep model predictive iterative (MMPI) method, incorporates the cell model in a model predictive fashion. Finally, a SOP verification methodology is presented that incorporates drive cycle data to realistically excite the battery model. Simulation results compare the proposed SOP algorithms to conventional approaches, where it is shown that higher accuracy can be achieved for larger time horizons.

Index Terms—Battery management systems, battery modeling, cell parameter variation, model predictive iterative, power estimation.

I. INTRODUCTION

BATTERIES are a fundamental component to enable clean, electrified, and sustainable transportation [1], [2]. Battery state estimation (BSE) plays a key role in a battery management system [3]–[5]. Two major functions of BSE are state-of-charge (SOC) [3]–[5] and state-of-power (SOP) estimation [8]–[20]; the latter reports peak power capability, e.g., maximum power for an arbitrary time interval. To accelerate the development of BSE algorithms and to facilitate the verification and validation effort, a model-in-the-loop (MIL) methodology is needed, which employs sufficient model fidelity with high-enough execution speed to perform practical simulation and design studies. The battery model should consider nonlinearities and time-varying effects that are typical of electrochemical cells to generate battery state and parameter variables for benchmarking BSE performance. For SOC verification, this is relatively straightforward since true SOC is an explicit state variable of the model that can be outputted to gauge SOC estimation performance. For SOP algorithm testing, this is not the case since calculation of peak power depends not only on the model’s current state/parameter values but on its future values and on the time horizon of interest as well. To properly assess the performance of a SOP algorithm, one must be able to generate a “true” SOP profile as a function of time; this can be done by determining the maximum sustained current allowed for some chosen time interval and multiplying by voltage. Therefore, peak power calculation can be simplified to peak current calculation. For verification of both SOC and SOP, a battery model and an SOP calculation method are needed for algorithm testing and development, e.g., as shown in Fig. 1. Aside from algorithm testing, the same SOP calculation method, or its simplified version, can be also directly part of SOP calculation within the battery management system software.

SOP estimation has been previously investigated using equivalent circuit models to calculate maximum current subject to cell limits such as voltage and SOC [8]–[20]. To improve voltage-limit-based SOP estimation, dynamic models considering, for example, resistor–capacitor (RC) pairs have been proposed [9], [10]. However, the variation of the circuit parameters such as open-circuit voltage (OCV), ohmic resistance, resistances, and capacitance values of RC pairs is usually neglected in the time horizon of interest. In [8], a multiparameter battery dynamic model is presented and a power estimation method is

![Fig. 1. Outputs for SOC and SOP algorithm benchmarking. LUT denotes lookup table. The input to the battery model is voltage V or current i.](http://ieeexplore.ieee.org/)

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evaluated that considers a first-order extrapolation of OCV. In [11], SOP estimation based on a dynamic model is employed, and a bisection method is used to find the charging/discharging current limits that account for the exact future value of the nonlinear OCV. In [12], joint SOC and SOP estimation using an adaptive extended Kalman filter is presented; SOP calculation in [12] also considers variation in the OCV. In [13], a review of state monitoring of lithium-ion (Li-ion) batteries is enumerated, including SOC, state of health, SOP, and remaining useful life. For SOP estimation, adaption of the model to the aging status of the battery, current dependence of the battery impedance, and multicell difference in the battery pack of power estimation methods are discussed. In [14], current dependence of battery impedance at lower temperature and aged batteries is evaluated and the method to estimate battery impedance is proposed. In [15], a novel approach to estimate power is proposed, which incorporates the current dependence into the nonlinear battery model. The proposed method shows improvement of power estimation at lower temperature and aged batteries. In [17], SOP estimation and SOF estimation based on a Kalman filter are presented. In [18], a new algorithm to estimate the maximum charge and discharge power values is proposed, which incorporates the diffusion effect. The diffusion effect is modeled as the nonlinear resistance in the equivalent circuit model of the battery, and the proposed method is validated by simulation results in a vehicle simulation environment. In [19], a model-based dynamic peak power method is presented for LiNMC and LiFePO4 batteries based on a linear-parameter-varying battery model. The Levenberg–Marquardt algorithm is applied to find the peak current and compared with the bisection method in [11] in terms of accuracy and computational complexity. The proposed method is validated in different operating conditions such as varying temperature and aging levels. In [20], a real-time prediction of peak power for Li-ion batteries is presented at different temperatures and aging conditions. A dual Kalman filter is utilized to estimate the present state/parameters of the batteries, and therefore, peak power estimation can be applicable to different temperatures and aging conditions. The method demonstrates good accuracy and adaptability through experiments.

For best accuracy, variations in all models parameters should be taken into account, particularly for larger time horizon intervals. In most previous publications, despite considering aging effect or current dependence of the battery, power is estimated at shorter time horizon intervals, and therefore, the variations of model parameters are not addressed. This paper incorporates the model variation into the power estimation, which is applicable at longer time horizons.

This paper presents a higher fidelity equivalent circuit model that is useful for both SOC and SOP algorithm testing and development. Asymmetric parameter values and an arbitrary number of RC pairs are considered. The use of asymmetric values for charge and discharge for all resistances and capacitance values increases the fidelity of the model. The model is implemented in a MATLAB Simulink environment; in particular, PLECS Blockset is used for its computational efficiency. Methods to parameterize this model are also described within an optimization-based parameter fitting context.

Methods to calculate SOP given a known model are presented where two novel approaches are proposed. The first method is a voltage-limit-based method where extrapolation of resistor values and OCV are considered. It incorporates first-order extrapolation of multiple equivalent circuit model parameters. The second method is a multistep model predictive iterative (MMPI) method. It updates the circuit parameters in smaller time intervals where constant or linear variations of the parameters are considered. Both work directly with the battery model presented in this paper, and both can be directly used in an online SOP calculation algorithm.

Lastly, a SOP verification methodology is described that incorporates drive cycle data to realistically excite the battery model. The same methodology can be used for hardware testing to experimentally validate SOP estimation and calculation performance. Simulation results in a MIL context are provided for SOP calculation and are assessed using the proposed SOP verification methodology.

The remainder of this paper is organized as follows. The battery model is presented in Section II. Parameterization and validation of this model are discussed in Section III. Section IV reviews conventional SOP peak current calculation methods. Section V presents the proposed SOP peak current calculation methods. The new SOP verification methodology and its simulation results comparing the proposed SOP peak current calculation methods are shown in Section VI, and the paper is concluded in Section VII.

II. BATTERY MODEL

An asymmetric parameter equivalent circuit model shown in Fig. 2 is employed. The parameter definitions are enumerated in the Appendix; they are functions of SOC and temperature. Columbic losses can be modeled as charging/discharging inefficiencies, which contribute to the difference between the amount of energy put into a cell and the energy extracted; therefore, the internal SOC state is updated according to the following formula to include the effect of charging/discharging inefficiencies:

$$\text{SOC}(t) = -\frac{1}{C_{\text{max}}} \int \left[ \eta_c i^- (t) + \eta_d^{-1} i^+ (t) \right] dt \quad (1)$$

where $i^- (t) = \min (i(t), 0)$, $i^+ (t) = \max (i(t), 0)$, $0 \leq \eta_c \leq 1$, and $0 \leq \eta_d \leq 1$ are the charging and discharging efficiencies.
The definition of cell capacity can impact the meaning of the efficiency parameters. For example, let $Ah_c$ be the (charging) capacity via current integration from 0% to 100% SOC and $Ah_d$ be the (discharging) capacity via current integration from 100% to 0% SOC. Inefficiencies result in $Ah_d < Ah_c$: this ratio can be extracted from experimental data. Now, consider the following scenarios for capacity definition. If $Ah_c$ is defined as the capacity, charging efficiency $\eta_c$ is one. Thus, the discharging efficiency $\eta_d$ is defined as

$$\eta_d = \frac{Ah_d}{Ah_c}. \quad (2)$$

Similarly, if $Ah_d$ is defined as the capacity, the inefficiencies are expressed as

$$\eta_c = \frac{Ah_d}{Ah_c}, \quad \eta_d = 1. \quad (3)$$

Assuming symmetric losses, charging efficiency and discharging efficiency can be redefined as

$$\eta_c = \eta_d = \sqrt{\frac{Ah_d}{Ah_c}}. \quad (4)$$

Using the preceding equation, the following definition for capacity is used in this paper:

$$C_{\text{max}} = \sqrt{Ah_c Ah_d}. \quad (5)$$

### III. Model Parameterization

The model can be parameterized using high pulse power characterization (HPPC) tests and OCV–SOC charge/discharge tests. This type of data comes in the form of voltage and current profiles over time, typically over the full SOC range and at multiple temperature points. Optimization-based fitting methods are used to extract parameters.

#### A. SOC–OCV Curve

The following equation is used for SOC–OCV fitting:

$$V_{oi} = k_0 - k_1/s_i + k_2 s_i + k_3 \log s_i - k_4 \log(1 - s_i) + k_5 s_i^2 + k_6 s_i^3 + k_7 s_i^4 + k_8 s_i^5 = s_i^T \mathbf{k} \quad (6)$$

where $V_{oi}$ is the $i$th OCV data point, and $s_i$ is SOC and is a number between zero and one. Variables $s$ and $\mathbf{k}$ consolidate the $s_i$ and $k_j$ terms into column vectors. The use of the higher order terms provides extra fidelity in the model. To ensure a nondecreasing fitted curve with these extra terms, the following derivative constraints are used:

$$\frac{dV_{o}}{dz_i} = k_1/s_i^2 + k_2 + k_3/s_i + k_4/(1 - s_i) + 2k_5 s_i + 3k_6 s_i^2 + 4k_7 s_i^3 + 5k_8 s_i^4 = d_i^T \mathbf{k} \geq 0. \quad (7)$$

#### B. Resistor and Capacitor Parameterization

HPPC data over the full SOC range is used in an optimization-based fitting routine to obtain resistor and capacitor values. The voltage profiles are preprocessed to remove the effect of OCV in the manner shown in Fig. 3. Since there is a large rest time prior to the pulse, it is assumed that the initial voltages of the RC states are zero.

Consider a current pulse of $i_{\text{pulse}}$ amperes of duration $t_{\text{pulse}}$ that starts at time $t = 0$, its ideal OCV subtracted voltage response can be derived as

$$V_{OS}(t) = \begin{cases} -\left(R_0 + \sum_{j=1}^{n} R_j (1-e^{-t_j/t_{\text{pulse}}}) \right) i_{\text{pulse}}, & t \leq t_{\text{pulse}} \\ -\sum_{j=1}^{n} R_j (1-e^{-t_{\text{pulse}}/t_{j}}) i_{\text{pulse}} e^{-t_{j}/t_{\text{pulse}}}, & t > t_{\text{pulse}} \end{cases} \quad (9)$$

where $R_i$, $i = 0..n$, and $t_j = R_j C_j$, $j = 1..n$, are parameters representing resistances and time constants, respectively. They are assumed to be constant over the pulse interval. Given voltage/current data and using (9), the following optimization can be formulated for parameter fitting:

$$\min_{\mathbf{r}, \tau} \sum_{k} (V_{\text{Data}}^{OS_k} - V_{\text{OS}}(\mathbf{r}, \tau, t_k))^2,$$

$$\mathbf{r} = [R_0, R_1 \cdots R_n], \quad \tau = [\tau_1 \tau_2 \cdots \tau_n]. \quad (10)$$
IV. Conventional Peak Current Calculation Methods

Here, battery peak power estimation [16]-[20] based on peak current calculation and the circuit model shown in Fig. 2 is described. Conventional methods of battery peak current estimation are reviewed and include SOC-limited, voltage-limited ohmic resistance (VLOR)-only, and voltage-limited first-order extrapolation of OCV (VLEO) with RC dynamics. The VLEO method is extended to an arbitrary number of RC pairs.

The maximum voltage when charging and the minimum voltage when discharging are denoted as \(V_{\text{max}}\) and \(V_{\text{min}}\), respectively. The maximum SOC and the minimum SOC are defined as \(s_{\text{max}}\) and \(s_{\text{min}}\), respectively. Without loss of generality, the subsequent derivations will consider the single-cell case. Moreover, for a cleaner presentation, the following variables are subsequently used such that, during charging, \(\zeta = \eta_c\), \(R_j = R_j^c\), \(\tau_j = \tau_j^c = R_j^c C_j^c\), and \(i_{\text{peak}} = i_{\text{min}} \leq 0\), and during discharging, \(\zeta = 1/\eta_d\), \(R_j = R_j^d\), \(\tau_j = \tau_j^d = R_j^d C_j^d\), and \(i_{\text{peak}} = i_{\text{max}} \geq 0\).

A. SOC-Limited Method

The SOC-limited method is used to estimate the battery peak current based on the minimum and maximum SOCs. The equation for SOC calculation is

\[
s(t + \Delta t) = s(t) - i_{\text{peak}} \frac{\Delta t}{C_{\text{max}}} \tag{11}
\]

where \(\zeta\) is the Coulomb efficiency factor, which is equal to \(\eta_c\) when charging and is equal to \(1/\eta_d\) when discharging, and \(C_{\text{max}}\) is the cell capacity. The peak (minimum) charging and peak (maximum) discharging currents by using the SOC-limited method are

\[
i_{\text{c, min}} = C_{\text{max}}(s - s_{\text{max}}) \eta_c \Delta t, \quad i_{\text{c, SOC}} = C_{\text{max}}(s - s_{\text{min}}) \eta_c^{-1} \Delta t. \tag{12}
\]

Note that the functional dependence on time has been dropped for a cleaner presentation.

B. VLOR Method

The VLOR method estimates the peak power based on a simplified circuit model, which only includes an OCV and internal resistance, i.e.,

\[
V(t + \Delta t) \approx V_o(s(t)) - R_0(s(t)) i_{\text{peak}} \tag{13}
\]

where \(V(t)\) is the terminal voltage of the cell at time \(t\); \(V_o(s(t))\) is the OCV at present SOC state \(s(t)\), and \(R_0\) is the charging or discharging internal resistance of the cell. The peak (minimum) charging and peak (maximum) discharging currents by using the VLOR method are defined as

\[
i_{\text{c, VLOR}} = \frac{V_o - V_{\text{max}}}{R_0}, \quad i_{\text{d, VLOR}} = \frac{V_o - V_{\text{min}}}{R_0} \tag{14}
\]

where \(R_0^c\) and \(R_0^d\) are the ohmic resistances for charging and discharging, respectively. Although simplistic, this estimate can be a reasonable initial guess for iterative algorithmic methods that perform peak current determination.
C. VLEO Method

The limitation of the VLEO method is that it neglects the transient dynamics of the battery. A voltage-limited extrapolation in the OCV method [16] that considers a first-order extrapolation of OCV and includes RC transients is described here; it is called the VLEO method. It considers the dynamics of the RC elements and OCV change over a specified time range. The voltage response $V_j$ of the $j$th RC pair assuming constant parameters during interval $\Delta t$ is derived as

$$V_j(t + \Delta t) = e^{-\Delta t/\tau_j} V_j(t) + R_j (1 - e^{-\Delta t/\tau_j}) i_{\text{peak}}$$

$j \in [1, n]$. (15)

A Taylor series expansion of $V_o$ is used to extrapolate OCV and is derived as

$$V_o(s + \Delta t) = V_o(s) - i_{\text{peak}} \frac{\Delta t}{C_{\text{max}}} \frac{\partial V_o}{\partial s} + \text{h.o.t.}$$

where “h.o.t.” denotes higher order terms that are subsequently neglected. Using (15) and (16), the voltage response of the cell can be calculated as

$$V(t + \Delta t) = V_o(s + \Delta t) - R_0(s + \Delta t) i_{\text{peak}} - \sum_{j=1}^{n} V_j(t + \Delta t) \approx V_o(s) - i_{\text{peak}} \frac{\Delta t}{C_{\text{max}}} \frac{\partial V_o}{\partial s} - R_0(s) i_{\text{peak}} - \sum_{j=1}^{n} e^{-\Delta t/\tau_j} V_j(t) + R_j (1 - e^{-\Delta t/\tau_j}) i_{\text{peak}}.$$  (17)

Rearranging the preceding equation, the peak charging and discharging currents can be readily solved as

$$i_{\text{c,VLEO}} = V_o - \sum_{j=1}^{n} e^{-\Delta t/\tau_j} V_j(t) - V_{\text{max}}$$

$$= \frac{\Delta t}{\Delta t} e^{-\Delta t/\tau_j} V_j(t) - V_{\text{max}}$$

$$i_{\text{d,VLEO}} = \frac{\Delta t}{\Delta t} e^{-\Delta t/\tau_j} V_j(t) - V_{\text{min}}$$

V. PROPOSED PEAK CURRENT CALCULATION METHODS

The conventional methods do not consider variations of all resistor and capacitor values during the constant current pulse $i_{\text{peak}}$. Since the resistor/capacitor parameters are functions of SOC, they can potentially vary greatly during the current pulse. Improvements to peak current calculation are described; they consider these variations in a model predictive fashion.

A. VLEO Method

An analytical voltage-limited extrapolation method is presented that considers RC elements, first-order extrapolation of all resistors, extrapolation of OCV, and constant RC time constants, i.e., voltage limited with extrapolations of resistances and OCV (VLERO) method. A Taylor series expansion of the variation of resistances is expressed as

$$R_j(s + \Delta t) = R_j(s) - i_{\text{peak}} \frac{\Delta t}{C_{\text{max}}} \frac{\partial R_j}{\partial s} + \text{h.o.t.}$$

Using the preceding equation with the h.o.t. neglected, the voltage response of the RC elements given in (15) is modified to

$$V_j(t + \Delta t) = e^{-\frac{\Delta t}{\tau_j}} V_j(t) + \int_{t}^{t+\Delta t} e^{-\frac{(s-t)}{\tau_j}} R_j(T) \frac{\partial R_j}{\partial s} \, dT \cdot i_{\text{peak}}, j \in [1, n]$$

Using (20) and (21), the voltage response in (15) or (17) is replaced with

$$V(t + \Delta t) \approx V_o - i_{\text{peak}} \frac{\Delta t}{C_{\text{max}}} \frac{\partial V_o}{\partial s} - \left( R_0 - i_{\text{peak}} \frac{\Delta t}{C_{\text{max}}} \frac{\partial R_0}{\partial s} \right) i_{\text{peak}} - \sum_{j=1}^{n} e^{-\Delta t/\tau_j} V_j(t) + R_j (1 - e^{-\Delta t/\tau_j}) i_{\text{peak}} - \sum_{j=1}^{n} \frac{\tau_j}{C_{\text{max}}} \frac{\partial R_j}{\partial s} \left( 1 - e^{-\frac{\Delta t}{\tau_j}} - \frac{\Delta t}{\tau_j} \right) i_{\text{peak}}^2.$$  (21)

This can be rearranged into a quadratic equation of the form

$$a_i i_{\text{peak}}^2 + b_i i_{\text{peak}} + c_i = 0,$$

where

$$a_i = \frac{\Delta t}{\tau_j} \frac{\partial R_j}{\partial s} - \sum_{j=1}^{n} \frac{\tau_j}{C_{\text{max}}} \frac{\partial R_j}{\partial s} \left( 1 - e^{-\frac{\Delta t}{\tau_j}} - \frac{\Delta t}{\tau_j} \right),$$

$$b_i = -\frac{\Delta t}{\tau_j} \frac{\partial V_o}{\partial s} - R_0 - \sum_{j=1}^{n} R_j \left( 1 - e^{-\frac{\Delta t}{\tau_j}} \right),$$

$$c_i = V_o - \sum_{j=1}^{n} e^{-\Delta t/\tau_j} V_j.$$  (23)

To obtain peak charging current $i_{\text{c,VLEO}}$, $V = V_{\text{min}}$ is used with (23). To obtain peak discharging current $i_{\text{d,VLEO}}$, $V = V_{\text{max}}$ is used with (23). The quadratic root formula can be applied in both cases to obtain closed-form solutions. To avoid complex roots related to nonrealistic extrapolation scenarios, a correction of the resistance derivatives is applied prior to using the quadratic root formula. Fig. 6 shows an example of resistance as a function of SOC where the actual slope is shown as a red solid line. Using this slope can lead to extrapolation to negative resistance values. A correction shown in Fig. 6 modifies the slope using the minimum resistance. The green dashed line shows the corrected slope used to obtain real analytical solutions with the VLERO method.
end-of-horizon RC voltage states. This inner stage performs steps of length $\Delta$ \textit{SOC} using (11) and calculates RC voltage states based on time inner stage accepts a peak current input and then increments depend on SOC. The MMPI method works in two stages. An model in the form of parameter lookup tables or functions that a limited time horizon. This approach incorporates a battery to better capture the variation of resistor/capacitor values over many small time steps comes at the price of greater computational cost. An alternative approach is to linearly interpolate parameters during the time step with endpoints, as shown in Fig. 8(b); each endpoint corresponds to a known different value of SOC during the time horizon of interest. A hybrid approach is also possible that assumes nonvarying RC time constants and first-order variations of resistor values similar to (20). A notable difference is that the derivative used would be calculated by using the SOC values at the endpoints of the time-step intervals, e.g., the slopes shown in Fig. 8(b). The hybrid approach would use (21) to update the RC voltage states in Fig. 7.

The more general approach of independently varying resistor and time-constant values is also possible. Consider (20) and the following Taylor series expansion of time constants:

$$\tau_j (s(t + \Delta t)) = \tau_j (s(t) - i_{\text{peak}} \frac{\Delta t}{C_{\text{max}}}) + \text{h.o.t.}$$

Neglecting the higher order terms in (20) and (24), the dynamics of the RC voltage can be expressed as

$$\frac{d V_j (t + \Delta t)}{d \Delta t} = -V_j + \left( R_j (s(t)) - i_{\text{peak}} \frac{\Delta t}{C_{\text{max}}} \frac{\partial R_j}{\partial s} \right) i_{\text{peak}} \frac{\Delta t}{\tau_j (s(t))} \frac{\partial \tau_j}{\partial s} \bigg|_{s=s(t)}.$$

The solution to the preceding equation is the following nonlinear function that has current as an input; it can be found using MATLAB “dsolve” and is expressed as follows:

$$V_j (t + t\Delta) = V_j (t) \frac{1}{\xi \left( \frac{\Delta t}{\tau_j (s(t)), \delta_{\tau_j}} \right)}$$

$$+ i_{\text{peak}} R_j (s(t)) \left( 1 - \frac{1}{\xi \left( \frac{\Delta t}{\tau_j (s(t)), \delta_{\tau_j}} \right)} \right) \frac{\Delta t}{\tau_j (s(t))} + 1 + \frac{1}{\xi \left( \frac{\Delta t}{\tau_j (s(t)), \delta_{\tau_j}} \right)} 1 + \delta_{\tau_j}.$$

**B. MMPI Methods**

For power limit prediction over larger time intervals, the extrapolation methods can poorly approximate the variations in OCV and ECM parameters. An MMPI method is proposed to better capture the variation of resistor/capacitor values over a limited time horizon. This approach incorporates a battery model in the form of parameter lookup tables or functions that depend on SOC. The MMPI method works in two stages. An inner stage accepts a peak current input and then increments SOC using (11) and calculates RC voltage states based on time steps of length $\Delta t/M$, where $M$ is the number of time-step divisions. This inner stage performs $M$ substeps to calculate end-of-horizon RC voltage states $V_j$; it then uses them with end-of-horizon SOC to output end-of-horizon cell voltage. Model parameters in the lookup tables are used throughout this inner stage. A flowchart of this stage is shown in Fig. 7.

Two methods can be employed to update the RC voltage states at each step of the inner stage. One method is to assume that the parameters are constant during the time interval of the substep and that it is essentially a zero-order-hold-type approach, and it is shown in Fig. 8(a). RC voltages would be updated using (15) with parameter values corresponding to the SOC at the beginning of each time step in the inner stage. For small time steps, this method can work well; however, using

**Fig. 6.** Resistance variation over SOC and slope corrections. The solid (black) curve is the cell resistance, the solid (red) line is the first-order extrapolation of resistance, the dotted (gray) line is the minimum resistance, and the dashed (green) line is a corrected slope extrapolation.

**Fig. 7.** Flowchart for inner stage multistep voltage calculation.

**Fig. 8.** Parameter approximations for multistep ($M = 4$) peak current calculation. (a) Constant parameters. (b) Piecewise linear parameters.
where

$$\xi \left( \frac{\Delta t}{\tau_j}, \delta \tau_j \right) = \left( 1 + \delta \tau_j \frac{\Delta t}{\tau_j} \right)^{\frac{1}{\tau_j}}$$

$$\delta R_j = -\xi \frac{i_{\text{peak}}}{C_{\text{max}}} \left( \frac{\partial R_j}{\partial s} \right)$$

$$\delta \tau_j = -\xi \frac{i_{\text{peak}}}{C_{\text{max}}} \left( \frac{\partial \tau_j}{\partial s} \right).$$

This expression can then be used in the RC voltage state updates in Fig. 7. It is worthy to note that, since

$$\lim_{n \to 0} (1 + nx)^{\frac{1}{x}} = e,$$

then

$$\xi \left( \frac{\Delta t}{\tau_j}, 0 \right) = e^{\frac{\Delta t}{\tau_j}}.$$  \hspace{1cm} (28)

It is also noted here that the approach of assuming linear variations in capacitance (instead of time constant) and resistance does not lead to a simple analytical expression for voltage analogous to (26); this arises since integration of nonelementary functions is encountered.

An outer stage in the MMPI method effectively performs a root-finding task to determine the peak current that matches voltage limits of $V_{\text{min}}$ or $V_{\text{max}}$. Well-known root finding algorithms such as bisection method [19], [23] and secant method [23] are some examples that can be applied. A simple approach is to iteratively increase/decrease peak current until it is within a tolerance to the voltage limit. A flowchart of this approach is shown in Fig. 9. The VLERO method is used to obtain an initial peak current value. If the corresponding terminal voltage obtained by the inner stage is within the tolerance $\delta V$ of the voltage limit (maximum voltage for charging or minimum voltage for discharging), the iteration ends. If the terminal voltage is not close enough, the estimated current values are either increased or decreased in steps $\delta i$ until the voltage is within the tolerance.

**VI. VERIFICATION METHODOLOGY FOR POWER ESTIMATION**

A verification methodology for power estimation is developed in this paper to evaluate different approaches for peak current-based SOP algorithms. Verification of power estimation is shown in Fig. 10. The estimated peak current obtained by power prediction block will be used to excite the battery model at predetermined times. Thus, excitation current for the battery model could be the peak charging (minimum) current, the driving cycle current, or the peak discharging (maximum) current. The minimum voltage is chosen as 2.8 V, and the maximum voltage is chosen as 4.1 V. The maximum discharging current is first calculated and then used to excite the battery model to observe whether it would discharge to 2.8 V in a specified time (e.g., 15 or 60 s). Then, UDDS drive cycle current is used to excite the battery, and minimum charging current is calculated based on the new state of the battery. Minimum charging current is also used to charge the battery to see whether it will charge to the maximum voltage.

The battery model has been implemented in MATLAB Simulink using PLECS add-on toolboxes. The authors have observed that PLECS is more computationally efficient then native Simscape toolbox. An Intel i5 2.5-GHz Apple computer with 4-GB RAM was used for MIL simulations. Results are shown at two relatively low temperatures of 0 °C and −20 °C where cell resistances are such that voltage-based limits are a relevant limiting factor for SOP.

To obtain a peak power estimate, the peak current estimate is multiplied by cell voltage. A conservative estimate can be obtained by using the minimum cell operating voltage, i.e.,

$$P = V_{\text{min}} i_{\text{peak}}.$$  \hspace{1cm} (29)

The focus here will be on the voltage-limited peak currents and comparisons between them. This is done here to avoid convoluting the results with limits such as absolute current limits and SOC-limited currents.

Fig. 11 shows simulation results of the peak current verification methodology using a 15-s peak power availability time horizon at a 0 °C modeled test condition. The (conventional) VLEO method with a 2RC model was used. At the low SOC range, power is incorrectly overestimated, resulting in undesirable undervoltage conditions. The MMPI method was employed for the same test condition, and the results are shown in Fig. 12; they show the elimination of the undervoltage responses and a correct peak current calculation. Similar test cases for a 60-s peak power availability horizon are shown in Figs. 13 and 14 using the VLEO and MMPI methods.
Fig. 11. Peak current prediction by using the VLEO method with 15-s time horizon at 0 °C.

Fig. 12. Peak current prediction by using the MMPI method with 15-s time horizon at 0 °C.

Fig. 13. Peak current prediction using the VLEO method with 60-s time horizon at 0 °C. respectively. The cell model parameter variations during the pulse lead to both under- and overestimates of peak current using the VLEO method. The MMPI method has near-perfect performance. Additional plots are shown at −20 °C for the 60-s SOP time horizon in Figs. 15 and 16. At this temperature, the voltage-limit-based peak currents are nearly the same magnitude at the drive cycle currents; this highlights the importance of accurate voltage-limit-based SOP algorithms. The conventional VLEO method results in both under/over cell voltage conditions; these are eliminated using the MMPI method.

A comprehensive analysis was performed between the VLEO method, the proposed VLERO method, and variations of the proposed MMPI approaches. The following percentage voltage error in reaching the voltage limit was used:

\[
\epsilon_{V}^{d} = \frac{|V_{t}^2 - V_{\text{min}}^2|}{V_{\text{min}}^2}, \quad \epsilon_{V}^{c} = \frac{|V_{t}^2 - V_{\text{max}}^2|}{V_{\text{max}}^2}
\]
where $err_d$ and $err_c$ represent the maximum discharging and charging errors of the SOP peak current pulse, $V_{\min}$ and $V_{\max}$ are the minimum/maximum cell voltage limits, and $V_t$ is the voltage at the end of the peak current pulse.

Tables I–IV summarize the results of the comparisons at $0\,^\circ C$ and $-20\,^\circ C$ test conditions where mean and maximum voltage percentage errors according to (28) are reported. Three variations of MMPI were employed, two of which used three substeps ($M=3$) to reduce the computational load of the algorithm. The results show that the VLERO method marginally improves performance over the VLEO method. The MMPI variants have similar near-perfect performance in most cases with maximum error of 1.54%. The usage of fewer subintervals ($M=3$) in the time horizon had minimal impact on accuracy. In principle, computational efficiency when using $M=3$ is expected to be 20 times greater compared with $M=60$. The usage of piecewise linear-based updates, e.g., Fig. 4(b), in MMPI generally improved when $M=3$ was used; for $M=60$, there was no difference between piecewise linear and constant parameter-based MMPI variants.

### VII. CONCLUDING REMARKS

An asymmetric parameter battery model and its parameterization that is amenable to timely BSE development and verification have been presented. A unique MIL verification methodology has been presented and employed to evaluate the proposed SOP algorithms. Novel SOP calculation methods have been proposed; one method provides relatively simple analytical expressions for peak current considering linear extrapolations of resistor parameter values and OCV. More accurate MMPI methods have been also presented that employ closed-form expressions of voltage response and multiple subinterval time steps to improve computational efficiency. The methods have been evaluated at relatively low-temperature scenarios where voltage-limit-based SOP is relevant. The proposed MMPI methods had the best accuracy, whereas the new VLERO method had marginal improvement.
APPENDIX

VARIABLE NOMENCLATURE

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Dependence</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C'_i$</td>
<td>f(T,SOC)</td>
<td>Charging Capacitance, $j=1..n$</td>
</tr>
<tr>
<td>$C'_d$</td>
<td>f(T,SOC)</td>
<td>Discharging Capacitance, $j=1..n$</td>
</tr>
<tr>
<td>$R'_c$</td>
<td>f(T,SOC)</td>
<td>Charging Resistance, $i=0..n$</td>
</tr>
<tr>
<td>$R'_d$</td>
<td>f(T,SOC)</td>
<td>Discharging Resistance, $i=0..n$</td>
</tr>
<tr>
<td>$C_{max}$</td>
<td>none</td>
<td>Capacity</td>
</tr>
<tr>
<td>$\eta_c$</td>
<td>none</td>
<td>Charging inefficiency $\in [0,1]$</td>
</tr>
<tr>
<td>$\eta_d$</td>
<td>none</td>
<td>Discharging inefficiency $\in [0,1]$</td>
</tr>
<tr>
<td>$V_e$</td>
<td>f(T,SOC)</td>
<td>Open circuit voltage</td>
</tr>
<tr>
<td>$V_i$</td>
<td>-</td>
<td>Voltage of $j^{th}$ RC state</td>
</tr>
<tr>
<td>$s(t), s_i$</td>
<td>none</td>
<td>Short form notation for SOC</td>
</tr>
<tr>
<td>$k_i$</td>
<td>none</td>
<td>Co-efficient for SOC-OCV function</td>
</tr>
<tr>
<td>$V_{OC}$</td>
<td>-</td>
<td>Cell voltage minus open circuit voltage</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>none</td>
<td>Columbic efficiency</td>
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<tr>
<td>$\tau, \tau'$</td>
<td>f(T,SOC)</td>
<td>RC time-constants</td>
</tr>
<tr>
<td>$i_{max}, i_{max}^d$</td>
<td>-</td>
<td>Peak pulse currents</td>
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<tr>
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<td>-</td>
<td>Min/max cell voltage</td>
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<tr>
<td>$\Delta t$</td>
<td>-</td>
<td>Power availability time horizon</td>
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<tr>
<td>$M$</td>
<td>-</td>
<td>Number of sub-steps in M-MPI</td>
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<tr>
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<td>-</td>
<td>Percentage error of discharge pulse</td>
</tr>
<tr>
<td>$err_c$</td>
<td>-</td>
<td>Percentage error of charge pulse</td>
</tr>
</tbody>
</table>

REFERENCES


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