A Digital PWM Current Controller for Switched Reluctance Motor Drives

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Abstract—In this paper, a PWM current controller for the switched reluctance motor drives is proposed and digitally implemented. Parameter adaption is employed to guarantee both fast dynamics and robustness of the proposed current controller. The relationship between the proposed controller and the conventional controllers including PI and dead-beat controller is also presented. An improved sampling method is designed to avoid PWM delay in the control loop. Simulation and experimental results show that the proposed controller keeps similar dynamic response and accuracy as hysteresis controller under various testing conditions. However, compared with the hysteresis controller, the proposed current controller needs much lower sampling rate and has a constant switching frequency.

Index Terms—Adaptive control, current control, motor drives, pulse width modulation (PWM), switched reluctance motor (SRM).

I. INTRODUCTION

SWITCHED reluctance motor (SRM) is an attractive alternative to the widely used induction and synchronous machines, due to its simple and low-cost structure, high reliability, and performance at high speeds [1]. However, an SRM suffers from its own drawbacks due to its double-salient structure, including high-torque ripple and acoustic noise. Conventionally, SRM is driven by asymmetric half bridges and there are three main control methods for SRM: single pulse operation, chopping-voltage PWM, and chopping-current regulation. Single pulse operation is usually used in high-speed operation. Chopping-voltage PWM method is equivalent to reduced dc voltage signal pulse operation. In order to reduce the torque ripple at low speed, chopping current regulation is generally used.

Fig. 1 shows a typical control diagram for SRM driven by asymmetric half bridges. Current controller is employed to generate switching signals for the asymmetric half bridges according to the current reference. The current reference is either given by a speed controller or a torque distributor. If the current reference comes directly from a speed controller, flat top chopping current for each phase is employed. Due to the strong nonlinearity, in some cases, the flat top chopping current regulation might not provide satisfactory performance. Therefore, torque sharing control is used to distribute torque production between two phases in order to produce constant torque [2]–[7].

Both flat top chopping current regulation and torque sharing control rely on accurate current controllers. Hysteresis control is one of the most popular current control schemes in SRMs, due to its fast dynamic response and model independency [4]–[8]. However, hysteresis controller also suffers from drawbacks including variable switching frequency and very high sampling rate [9]–[11]. Variable switching frequency in hysteresis control makes it difficult to design the electromagnetic interference (EMI) filter and may cause an acoustical noise. High-speed ADCs have higher sampling rate, however, they add additional cost to the SRM drive system.

In order to avoid the drawbacks of the hysteresis current controller, fixed frequency PWM controllers have been studied [9], [11]–[16]. In [12], an open loop PWM controller is used, whereas in [9], a proportional-integral (PI) current controller has been investigated and a current sampling method for digital control have been introduced. A proportional (P) controller with an iterative learning control is proposed in [17] to achieve accurate current control. In [11], [13]–[16], back EMF compensation to the PI current controller has been analyzed. In [11], the gains of the PI controller are adjusted according to current and rotor position. However, a PI controller suffers from either slow response or possible overshoot. It is also difficult to tune the PI controller in SRM applications due to the highly nonlinear characteristics of the machine.

Model-based dead-beat flux controller are proposed in [18]–[21]. The dead-beat controller achieves constant switching frequency and lower sampling rate, while maintaining the similar dynamic response as hysteresis controller. However, the performance of a dead-beat controller relies on an accurate model and a large gain, which may degrade the performance of the dead-beat controller.
In [22], a Lyapunov function-based controller is proposed to solve model mismatch issue. The tracking error is bounded by the parameters of the controller. A sliding mode current controller is proposed in [23]. Parameters of these controllers are carefully selected according to the model mismatch. These control methods need to store several look-up tables, which increase the storage and computational burden of the digital controller.

A digital PWM current controller for the SRM drives is proposed in this paper in order to achieve fast response, accurate tracking, immunity to noise, model mismatch, and stability. The proposed controller takes full advantage of the model information. Smaller feedback gain could be chosen in order to reduce noise sensitivity without degrading the performance. Parameter adaptation is adopted to deal with the model mismatch. Relationships between the proposed controller and the previously mentioned PI dead-beat controllers are discussed. Both the simulation and experimental results are provided to verify the performance of the proposed current controller.

II. MODEL OF SRM

By neglecting mutual coupling between phases, the phase voltage equation of an SRM can be given as

\[
\frac{du_w}{dt} = R_w \cdot i + \frac{d\psi(\theta, i)}{dt}
\]

where \(u_w\) is the phase voltage applied on the phase winding, \(R_w\) is the winding resistance, \(\psi\) is the flux linkage, \(\theta\) is the rotor position, and \(i\) is the phase current.

Due to its double salient structure and saturation, \(\psi\) is a non-linear function of both \(i\) and \(\theta\). Fig. 2 shows the measured flux linkage profile of the SRM studied in this paper. The rotor spins \(360^\circ\) per electric period. The aligned positions are \(0^\circ\) and \(360^\circ\). The unaligned position is \(180^\circ\). Fig. 2 could be stored into a lookup table when digital control is applied.

Considering the modeling errors, the real flux linkage is represented as

\[
\psi(\theta, i) = \alpha \psi_m(\theta, i)
\]

where \(\psi_m\) is the modeled flux linkage profile used in the controller, and factor \(\alpha\) is a positive number that donates the relationship between the modeled flux linkage profile and the real one.

In the ideal case, the modeled flux linkage profile exactly matches the real one, and \(\alpha = 1\). But in practice, no matter whether \(\psi_m\) is obtained by the experimental measurement or by an FEA calculation, there may be some mismatch between \(\psi\) and \(\psi_m\). In this case, \(\psi_m\) is unknown and \(\alpha\) may be variable and there is

\[
\alpha > 0
\]

\[
\alpha - B_{\alpha} \leq \alpha \leq \alpha + B_{\alpha}
\]

\[
|\dot{\alpha}| \leq B_{\dot{\alpha}}
\]

(3)

where \(\bar{\alpha}\) is the average value of \(\alpha\), \(B_{\alpha} \geq 0\) is the variation bound of \(\alpha\), and \(B_{\dot{\alpha}} \geq 0\) is the maximum variation rate of \(\alpha\). The values of \(B_{\alpha}\) and \(B_{\dot{\alpha}}\) depend on the modeling errors of the studied motor.

Considering the resistances and voltage drops on windings and switches, the phase voltage equation could be written as

\[
\frac{d\psi_m(\theta, i)}{dt} = -\frac{1}{\alpha} \left( R_w + R_c \right) i - \frac{1}{\alpha} \left( \psi_m(\theta, i) \frac{d\alpha}{dt} + v_c + v_m + v_n \right) + \frac{1}{\alpha} u_c
\]

\[
\frac{d\psi_m(\theta, i)}{dt} = -\frac{1}{\alpha} Ri - \frac{1}{\alpha} v + \frac{1}{\alpha} u_c
\]

\[
R = R_w + R_c
\]

\[
v = \psi_m(\theta, i) \frac{d\alpha}{dt} + v_c + v_m + v_n
\]

(4)

where \(u_c\) donates the converter output voltage, \(R_c\) donates the equivalent resistance of the converter, \(R_w\) could be obtained from either experiments or data sheets, but it changes according to current, temperature, gate source (GS) voltage, etc. \(v_c\) donates the voltage drop on the converter, \(v_m\) donates the voltage drop caused by mutual inductance, \(v_n\) reflects all other voltage drops, and noises in the system. Equation (4) could be formulated as

\[
\frac{d\psi_m(\theta, i)}{dt} = -\frac{1}{\alpha} Ri - \frac{1}{\alpha} v + \frac{1}{\alpha} u_c
\]

\[
R = R_w + R_c
\]

\[
v = \psi_m(\theta, i) \frac{d\alpha}{dt} + v_c + v_m + v_n
\]

(5)

where \(R\) is the total equivalent resistance and \(v\) is the total equivalent voltage drop. They are uncertain parameters that are not easy to model. The values of \(R\) and \(v\) are both unknown and may be variable, which are represented as

\[
Ra > 0
\]

\[
\bar{R} - B_{R} \leq R \leq \bar{R} + B_{R}
\]

\[
|\bar{R}| \leq B_{\bar{R}}
\]

\[
\bar{v} - B_{v} \leq v \leq \bar{v} + B_{v}
\]

\[
|\bar{v}| \leq B_{\bar{v}}
\]

(6)

where \(R\) donates the average value of \(R\), \(B_{R} \geq 0\) donates the variation bound of \(R\), and \(B_{\bar{R}} \geq 0\) donates the maximum variation rate of \(R\). \(R\) is also positive, \(\bar{v}\) donates the average value of \(v\), \(B_{v} \geq 0\) donates the variation bound of \(v\), and \(B_{\bar{v}} \geq 0\) donates the maximum variation rate of \(v\).
III. PROPOSED CURRENT CONTROLLER

A current controller can either control the current directly or control the current indirectly by controlling the flux linkage. As shown in Fig. 2, for a certain position $\theta$, $\psi$ is a monotone increasing function of $i$. For any $i_1 \geq 0$, $i_2 \geq 0$ there is

\[
\psi_m(\theta, i_1) > \psi_m(\theta, i_2) \iff i_1 > i_2 \\
\psi_m(\theta, i_1) = \psi_m(\theta, i_2) \iff i_1 = i_2 \\
\psi_m(\theta, i_1) < \psi_m(\theta, i_2) \iff i_1 < i_2.
\]

(7)

Therefore, the phase current can be controlled by controlling its corresponding flux linkage. The SRM model shown in (4) contains unknown parameters, a current controller with estimated parameter values could be constructed as

\[
u_c = \alpha \frac{d\psi_m(\theta, i_{ref})}{dt} + \dot{R} \dot{i}_{ref} + \dot{v} + ke
\]

(8)

where $\psi_m(\theta, i_{ref})$ is the reference flux linkage calculated by the reference current $i_{ref}$ and rotor position $\theta$, $\dot{\alpha}$ is the estimated value of $\alpha$, $\dot{R}$ is the estimated value of $R$, $\dot{v}$ is the estimated value of $v$, $k$ is a positive constant, and $e$ is the flux linkage error which can be expressed as

\[
e = \psi_m(\theta, i_{ref}) - \psi_m(\theta, i).
\]

(9)

Substituting (8) into (5), the flux linkage error dynamics can be derived as

\[
\begin{align*}
\dot{e} &= -\frac{k}{\alpha} e - \dot{R} \dot{e}_i + \frac{1}{\alpha} \dot{\alpha} \psi_m(\theta, i_{ref}) + \frac{1}{\alpha} \dot{R} \dot{i} + \frac{1}{\alpha} \dot{v} \\
\dot{\alpha} &= \alpha - \dot{\alpha} \\
\dot{R} &= R - \dot{R} \\
\dot{v} &= v - \dot{v} \\
e_i &= i_{ref} - i
\end{align*}
\]

(10)

where $\dot{\alpha}$, $\dot{R}$, and $\dot{v}$ are the estimation errors, $e_i$ is the current error.

If a Lyapunov candidate is selected as

\[
V = \frac{1}{2} e^2 + \frac{1}{2\alpha k_{\alpha}} \dot{\alpha}^2 + \frac{1}{2\alpha k_{R}} \dot{R}^2 + \frac{1}{2\alpha k_{v}} \dot{v}^2
\]

(11)

where $k_{\alpha}$, $k_R$, and $k_v$ are positive constants. Then, the derivative of the Lyapunov candidate is

\[
\dot{V} = -\frac{k}{\alpha} e^2 - \frac{\dot{R}}{\alpha} e e_i + \frac{1}{\alpha k_{\alpha}} \dot{\alpha} \dot{\alpha} \\
+ \frac{1}{\alpha} \dot{\alpha} \psi_m(\theta, i_{ref}) e - \frac{1}{\alpha k_{\alpha}} \dot{\alpha} \dot{\alpha} \\
+ \frac{1}{\alpha} \dot{R} e - \frac{1}{\alpha k_{R}} \dot{R} \dot{R} \\
+ \frac{1}{\alpha} \dot{v} e - \frac{1}{\alpha k_{v}} \dot{v} \dot{v} \\
+ \frac{\dot{\alpha} \dot{\alpha}}{\alpha k_{\alpha}} + \frac{\dot{R} \dot{R}}{\alpha k_{R}} + \frac{\dot{v} \dot{v}}{\alpha k_{v}}.
\]

(12)

It can be seen from (12) that if $\dot{\alpha}$, $\dot{R}$, and $\dot{v}$ are chosen as

\[
\begin{align*}
\dot{\alpha} &= k_{\alpha} \psi_m(\theta, i_{ref}) e \\
\dot{R} &= k_{R} R e \\
\dot{v} &= k_{v} e
\end{align*}
\]

(13)

Then, (12) becomes

\[
\dot{V} = -\frac{k}{\alpha} e^2 - \frac{\dot{R}}{\alpha} e e_i
\]

(14)

A. Constant Parameters

$k$, $R$, and $\alpha$ are positive constants, and according to (7), $e_i$ and $e$ have the same sign. If $\alpha$, $R$, $v$ are constant, i.e.,

\[
\dot{\alpha} = 0, \dot{R} = 0, \dot{v} = 0
\]

(15)

$\dot{V}$ becomes

\[
\dot{V} = -\frac{k}{\alpha} e^2 - \frac{\dot{R}}{\alpha} e e_i
\]

(16)

Therefore, $\dot{V}$ is seminegative definite. This indicates that the system is globally asymptotically stable, and $e$ is going to converge to zero. If $e$ converges to zero, the system is internally stable. The convergence rate of $e$ is determined by $k$ and $\alpha$. Since $\alpha$ is around 1, $k$ could be selected to adjust the convergence rate. According to (16), a larger $k$ gives a faster convergence rate, which means faster dynamic response. However, according to (8), $k$ is the feedback gain of the error, in this case, a large gain means that the controller is more sensitive to noise. Therefore, the selection of $k$ is a tradeoff between the dynamic response and robustness. According to (10), if $e$ converges to zero, for any $\psi_m(\theta, i_{ref})$ and $i$, there will be

\[
\dot{\alpha} \psi_m(\theta, i_{ref}) + \dot{R} i + \dot{v} = 0.
\]

(17)

As is known, the adaptive controllers suffer from parameter drafting. Since all the estimated parameters are bounded, the controller will be stable. However, parameters will not necessarily converge to their real values unless persistent excitation condition is satisfied [24]. For the case of (13), $\psi_m(\theta, i_{ref})$ and $i$ need to be “rich” enough to guarantee the convergence. Fig. 3 shows a typical waveform of $\psi_m(\theta, i_{ref})$ and $i_{ref}$ with flat-top current control. It can be seen that $i_{ref}$ is a constant number and $i_{ref}$ is zero, while $\psi_m(\theta, i_{ref})$ is a nonlinear function of time.
The nonlinearity of $\psi_m(\theta, i_{\text{ref}})$ will provide sufficient frequencies to make $\psi_m(\theta, i_{\text{ref}})$ “rich.” This is another reason why flux linkage is selected to be controlled instead of current. In this case, there is

$$
\Psi(t) = \left[ \dot{\psi}_m(\theta, i_{\text{ref}}, t), 1 \right]^T
$$

$$
\int_{t_0}^{t_\tau} \Psi(\tau) \Psi(\tau)^T d\tau \geq \gamma I
$$

where $t_0$ is the beginning of each stroke and $t_\tau$ is the end of the stroke. Equation (18) indicates that $\Psi(t)$ satisfies the exciting condition, which means $\|[\dot{\alpha}, \dot{v}]\|_2$ is going to converge per stroke [24]. In this case, as the controller is active each stroke, the estimation errors are going to converge to zero eventually and there will be

$$
\dot{\alpha} = \alpha \\
\dot{v} = v.
$$

However, if the flat-top current control is applied, $i$ may not be rich enough to guarantee the convergence of $\dot{R}$. In this case, a dead zone should be added to prevent parameter drafting of $\dot{R}$.

**B. Variable Parameters**

Practically, the parameters $\alpha$, $R$, and $v$ are not constant

$$
\dot{\alpha} \neq 0, \dot{R} \neq 0, \dot{v} \neq 0.
$$

Since $\alpha$, $R$, and $v$ have their own bounds, the adaption law in (13) should be modified by

$$
\dot{\alpha} = \begin{cases} 
-k_\alpha \dot{\psi}_m(\theta, i_{\text{ref}}) e, & \dot{\alpha} \in [\alpha - B_\alpha, \alpha + B_\alpha] \\
-k_\alpha \dot{\psi}_m(\theta, i_{\text{ref}}) e, & \dot{\alpha} > \alpha + B_\alpha \text{ and } \dot{\psi}_m(\theta, i_{\text{ref}}) e < 0 \\
k_\alpha \dot{\psi}_m(\theta, i_{\text{ref}}) e, & \dot{\alpha} < \alpha - B_\alpha \text{ and } \dot{\psi}_m(\theta, i_{\text{ref}}) e > 0 \\
0, & \text{else}
\end{cases}
$$

$$
\dot{R} = \begin{cases} 
k_\beta \dot{e}, & \dot{R} \in [\dot{R} - B_R, \dot{R} + B_R] \\
k_\beta \dot{e}, & \dot{R} > \dot{R} + B_R \text{ and } ie < 0 \\
k_\beta \dot{e}, & \dot{R} < \dot{R} - B_R \text{ and } ie > 0 \\
0, & \text{else}
\end{cases}
$$

$$
\dot{v} = \begin{cases} 
k_\varepsilon \dot{e}, & \dot{v} \in [\dot{v} - B_v, \dot{v} + B_v] \\
k_\varepsilon \dot{e}, & \dot{v} > \dot{v} + B_v \text{ and } e < 0 \\
k_\varepsilon \dot{e}, & \dot{v} < \dot{v} - B_v \text{ and } e > 0 \\
0, & \text{else}
\end{cases}
$$

This modification does not affect the system stability if the real values of $\alpha$, $R$, and $v$ do not exceed their bounds. At the same time, (22) defines the bounds of parameter estimation error

$$
|\dot{\alpha}| \leq 2B_\alpha, |\dot{R}| \leq 2B_R, |\dot{v}| \leq 2B_v.
$$

Combined with (3) and (6), there are

$$
\frac{\dot{\alpha}^2}{2\alpha k_\alpha} + \frac{\ddot{\alpha}}{2\alpha k_\alpha} + \frac{\ddot{v}^2}{2\alpha k_v} \leq 2B_\alpha + 2B_R + 2B_v = M
$$

$$
\frac{\dot{\alpha}}{\alpha k_\alpha} + \frac{\ddot{R}}{\alpha k_R} + \frac{\ddot{v}}{\alpha k_v} \leq 2B_\alpha B_R + 2B_R B_v + 2B_v B_v
$$

where $B_\alpha$, $B_R$, and $B_v$ are the bounds of $\dot{\alpha}$, $\dot{R}$, and $\dot{v}$, respectively. According to (14) and (11), there is

$$
e^2 = 2V - \frac{\dot{\alpha}^2}{\alpha k_\alpha} - \frac{\ddot{R}}{\alpha k_R} - \frac{\ddot{v}^2}{\alpha k_v}
$$

$$
V = -2k_\beta \dot{R} \left( V - \frac{\dot{\alpha}^2}{2\alpha k_\alpha} - \frac{\ddot{R}^2}{2\alpha k_R} - \frac{\ddot{v}^2}{2\alpha k_v} \right)
$$

$$
+ \frac{\dot{\alpha}}{\alpha k_\alpha} + \frac{\ddot{R}}{\alpha k_R} + \frac{\ddot{v}}{\alpha k_v}
$$

where $k_i > 0$ donates the relationship between $e$ and $e_i$. According to (25), if $V$ exceeds $\alpha N/2(k + \dot{R}) + M$, $V$ will be negative, and $V$ is going to decrease. Thus, the control error is bounded by

$$
|e| \leq \sqrt{\alpha N/2(k + \dot{R})}
$$

$$
= \sqrt{\frac{2B_\alpha B_R}{(k + \dot{R}) k_R} + \frac{2B_R B_v}{(k + \dot{R}) k_v}}.
$$

As shown from (26), for the predefined bounds and maximum variation rates of the unknown parameter, the control error is limited by $k_i, k_\alpha, k_R$, and $k_v$.

**IV. DIGITAL IMPLEMENTATION OF PROPOSED CURRENT CONTROLLER**

In digital implementation, the discrete form of (8) and (21) can be reformulated as

$$
\Delta \psi_m(k) = \psi_m(\theta(k + 1), i_{\text{ref}}(k + 1)) - \psi_m(\theta(k), i_{\text{ref}}(k))
$$

$$
u_c(k) = \frac{\dot{\alpha}(k)\Delta \psi_m(k)}{T} + \dot{R}(k) i_{\text{ref}}(k) + \dot{v}(k) + ke(k)
$$
where $T$ is the digital sampling time, $\theta(k+1) = \theta(k) + \omega T$, and $\omega$ is the electric angular speed of the SRM.

\[
\dot{\alpha}(k+1)' = \dot{\alpha}(k) + k_\alpha \Delta \psi_m(k) e(k)
\]

\[
\dot{\alpha}(k+1) = \begin{cases} 
\dot{\alpha}(k+1)' , & \dot{\alpha}(k+1)' \in [\alpha - B_\alpha, \alpha + B_\alpha] \\
\dot{\alpha} + B_\alpha, & \dot{\alpha}(k+1)' > \alpha + B_\alpha \\
\dot{\alpha} - B_\alpha, & \dot{\alpha}(k+1)' < \alpha - B_\alpha
\end{cases}
\]

\[
e(k)' = \begin{cases} 
e(k) , & |e(k)| > B_{DZ} \\
0 , & |e(k)| \leq B_{DZ}
\end{cases}
\]

\[
\dot{R}(k+1)' = \dot{R}(k) + k_R i(k) e(k) T
\]

\[
\dot{R}(k+1) = \begin{cases} 
\dot{R}(k+1)' , & \dot{R}(k+1)' \in [\dot{R} - B_R, \dot{R} + B_R] \\
\dot{R} + B_R, & \dot{R}(k+1)' > \dot{R} + B_R \\
\dot{R} - B_R, & \dot{R}(k+1)' < \dot{R} - B_R
\end{cases}
\]

\[
\dot{v}(k+1)' = \dot{v}(k) + k_v e(k) T
\]

\[
\dot{v}(k+1) = \begin{cases} 
\dot{v}(k+1)' , & \dot{v}(k+1)' \in [\bar{v} - B_v, \bar{v} + B_v] \\
\bar{v} + B_v, & \dot{v}(k+1)' > \bar{v} + B_v \\
\bar{v} - B_v, & \dot{v}(k+1)' < \bar{v} - B_v
\end{cases}
\]

where $\Delta \psi_m(k)$ is defined in (27). $B_{DZ}$ is the error dead zone, $\dot{\alpha}$, $\dot{R}$, and $\dot{v}$ are the estimated average values of $\alpha$, $R$, and $v$, respectively.

### A. PWM Delay Compensation

Fig. 4 shows the PWM modulation for digital control. The duty ratio is either obtained by $u_c/U_{DC}$ for soft chopping or $0.5 + 0.5(u_c/U_{DC})$ for hard chopping. $U_{DC}$ is the dc bus voltage. In the $k$th control period, current should be sampled at $t(k)$. But in practice, especially in a DSP control, if current is sampled at $t(k)$, it will take some time for the controller to calculate the duty ratio and the duty ratio for $t(k)$ is actually loaded into the PWM modulator at $t(k+1)$. This brings one sampling time delay into the control loop. In this case, the duty ratio for $t(k)$ should be calculated before $t(k)$. Mohamed and El-Saadany [10] propose a predictive current controller to solve the problem. However, the predictive current controller needs accurate model and increases the calculation burden for DSP, especially for nonlinear systems such as SRMs. Blaabjerg et al. [9] recommends that current should be sampled at $t(k-1/2)$, which means $i(k)$ is approximated by

\[
i(k) \approx i(k-1/2).
\]

As shown in Fig. 4, there is no switching action at $t(k-1/2)$, EMI noise at that instance can be avoided. Furthermore, the duty ratio can be calculated within half of the period and delay in the control loop is avoided.

The estimation of (29) is accurate if the average current of each $k$th period stays the same, as the $(k-1)$th period shown in Fig. 4. If average current between each period changes, as the $k$th period shown in Fig. 4, (29) is not accurate.

As shown in Fig. 4, with the symmetrical modulation, the voltage waveforms of the former half period and the latter half period are symmetric. Therefore, the flux could be estimated instead of current. The flux $\psi_m(\theta(k), i(k))$ could be approximated by

\[
\psi_m(\theta(k), i(k)) \approx \psi_m(\theta(k - 1/2), i(k - 1/2)) - \psi_m(\theta(k - 1), i(k - 1)).
\]

In (30), current is sampled at both $t(k-1/2)$ and $t(k-1)$, which doubles the sampling rate. The ADCs used in motor control is capable of working at the sampling rate of twice of the PWM frequency without increasing any cost. Similar to (29), (30) also avoids the EMI noise caused by the switching action, provides half control period for duty ratio calculation, and avoids the delay in the control loop as well.

Since the current sampling, and other calculations are performed at $t(k-1/2)$, the rotor position also has to be approximated with the information at $t(k-1/2)$. Fig. 5 shows the approximation of $\psi_m(\theta(k), i(k))$ and $\theta(k+1)$ for further use.

### B. Flux Reference Adjustment

When implemented in a digital processor, the current controller has to meet physical limits. Normally, when a phase is turned ON, the phase current is expected to rise quickly to the reference value, however, the voltage applied on the phase is limited by $U_{DC}$. It is necessary to adjust $\psi_m(\theta(k+1),$ $i_{ref}(k+1))$...
so that \( u_c(k) \) would not exceed \( U_{DC} \)

\[
\Delta \psi_m(k) = \psi_m(\theta(k+1), i_{ref}(k+1)) - \psi_{adj}(\theta(k), i_{ref}(k)) \]
\[
\Delta \psi_{P\max}(k) = \frac{T}{\hat{\psi}(k)} \left( U_{DC} - \hat{R}(k) i_{ref}(k) - \hat{v}(k) - ke(k) \right) \]
\[
\Delta \psi_{N\max}(k) = \frac{T}{\hat{\psi}(k)} \left( -U_{DC} - \hat{R}(k) i_{ref}(k) - \hat{v}(k) - ke(k) \right) \]
\[
\psi_{adj}(\theta(k+1), i_{ref}(k+1)) = \psi_{adj}(\theta(k), i_{ref}(k)) + \left\{ \begin{array}{l}
\Delta \psi_{P\max}(k) > \Delta \psi_{N\max}(k) \\
\Delta \psi_{P\max}(k) < \Delta \psi_{N\max}(k) \\
\Delta \psi_{N\max}(k) < \Delta \psi_{N\max}(k) \leq \Delta \psi_{P\max}(k).
\end{array} \right.
\] (31)

Therefore, \( \psi_m(\theta(k+1), i_{ref}(k+1)) \) and \( \psi_m(\theta(k), i_{ref}(k)) \) in (27) should be replaced by \( \psi_{adj}(\theta(k+1), i_{ref}(k+1)) \) and \( \psi_{adj}(\theta(k), i_{ref}(k)) \), respectively. Fig. 6 shows the procedure of calculating \( \psi_{adj}(\theta(k+1), i_{ref}(k+1)) \) and \( \psi_{adj}(\theta(k), i_{ref}(k)) \) according to (31). Fig. 7 shows the procedure of calculating \( e, \hat{\alpha}(k), \hat{R}(k), \) and \( \hat{v}(k) \) according to (28). Fig. 8 shows the procedure of calculating \( u_c(k) \) according to (27).

C. Relationship With Previously Proposed Controllers

As shown in Fig. 8, the controller of (27) consists of two parts: the feedback part and the feedforward part. The feedback part is sensitive to noise, while the feedforward part is immune to noise. In order to enhance the robustness of the controller, the feedforward part should give out most part of \( u_c \) so that less control effort is needed by the feedback part.

The digital controller of (27) has similar form with previously proposed controllers. For example, all the estimated parameters are taken as its real value, and \( k = 1/T \), then (27) becomes

\[
u_c(k) = \frac{\psi_m(\theta(k+1), i_{ref}(k+1)) - \psi_m(\theta(k), i(k))}{T} + \hat{R}i(k) + v.
\] (32)

This is a typical dead-beat controller proposed in [18]–[21]. If \( \hat{\alpha} \) is fixed as \( K_p \cdot T \) and only the adaption of \( \hat{v} \) is active with a gain of \( K_i \), then (27) becomes a PI controller

\[
u_c(k) = K_p \left( \psi_m(\theta(k+1), i_{ref}(k+1)) - \psi_m(\theta(k), i(k)) \right) + \hat{R}i(k) + \hat{v}(k)
\]
\[
\hat{v}(k+1) = \hat{v}(k) + K_i e(k)T.
\] (33)

From this point of view, the proposed controller could be regarded as the improvement of some of the existing controllers.
D. Parameter Selection

With the digital controller in (27), the error transfer function (10) could be rewritten in discrete domain as

\[
e(k+2) = \left(1 - \frac{k}{\alpha} T\right) e(k+1) - \hat{R}(k+1) T \dot{e}(k+1) + \frac{1}{\alpha} \hat{\alpha}(k+1) \Delta \psi_{\text{adj}} (\theta(k+1), i_{\text{ref}}(k+1)) + \frac{1}{\alpha} T \hat{R}(k+1) i(k+1) + \frac{1}{\alpha} T \hat{v}(k+1).
\]

(34)

Since the sampling time T is usually small enough, substituting (28) into (34), the error dynamics can be obtained as

\[
e z^2 - \left(1 - \frac{k}{\alpha} T\right) ez + Pe = O
\]

\[
P = k_a \frac{\Delta \psi_{\text{adj}} (\theta(k), i_{\text{ref}}(k)) \Delta \psi_{\text{adj}} (\theta(k+1), i_{\text{ref}}(k+1))}{\alpha} + k_R \frac{i(k+1)i(k)}{\alpha} T^2 + k_{\bar{v}} \frac{1}{\alpha} T^2
\]

(35)

where O is small enough bounded items, which could be taken as input of the error dynamic. The poles of the discrete transfer function of (35) are

\[
\lambda_{1,2} = \left(1 - \frac{k}{\alpha} T\right) \pm \sqrt{(1 - \frac{k}{\alpha} T)^2 - 4P}.
\]

(36)

To stabilize the system, the poles should be placed inside the unit cycle, and hence the limits of the parameters are

\[
0 < \frac{k}{\alpha} T < 2 + P
\]

\[
0 < P < \frac{1}{4}.
\]

(37)

It can be seen that in (32), k is selected to be 1/T and P is selected to be zero, and therefore, the poles are placed at zero. Due to the feedforward part in the proposed controller, a smaller k could be chosen. After k is chosen, k_a, k_R, and k_{\bar{v}} are selected to ensure the stability.

V. SIMULATION AND EXPERIMENTAL RESULTS

To verify the effectiveness of the proposed current controller for SRM, both the simulations and experiments are conducted. The flux profile of the studied SRM is shown in Fig. 2. Other parameters of the studied SRM are shown in Table I. To investigate the performance of the proposed controller in practical application, a linear torque sharing distributor is adopted to generate reference current for the current controller to produce the rated torque (3.2 N·m). To verify the performance of parameter adaption, the initial value of \(\bar{\alpha}\) is set to 0.5. Parameters of the current controller for both simulation and experiments are shown in Table II. Center-aligned PWM and bipolar modulation are adopted. Parameters of the experimental plant for both simulation and experiments are shown in Table III.

A. Simulation Results

Simulation (MATLAB/Simulink) model is built to verify the effectiveness of the proposed current controller for SRM. First, the proposed current controller is tested at low speed. The SRM is controlled to run at an mechanical speed of 1000 r/min (8000 r/min in electric speed). Fig. 9 shows the phase current waveform and its reference. As a comparison, hysteresis current control with hysteresis band of \(\pm 0.5\) A is applied on the same SRM model. The sampling rate of the hysteresis controller is set to 100 kHz. Since the hysteresis controller is digitally implemented, one-sample-time delay is taken into account. Fig. 10 shows the phase current and its reference. It is shown that the proposed current controller has almost the same dynamic response and accuracy compared with hysteresis controller. Even though the hysteresis current controller has a very narrow hysteresis band and very high sampling rate, due to the finite
sampling rate and the one-sample-time delay, the current ripples are still larger than that of the proposed controller.

Fig. 11 shows the calculated $u_c$ by the proposed controller. It is shown that due to the nonlinearity of SRM, $u_c$ is very nonlinear. This also indicates the proposed controller has very high bandwidth.

Fig. 12 shows the waveforms of control error($e$), $\hat{\alpha}$, $\hat{R}$, and $\hat{v}$. It is shown that the control error is large due to the mismatch of parameters in the beginning. After the convergence of the parameters, the control error is greatly reduced.

Fig. 13 shows the results with a conventional dead-beat controller when $\alpha = 1$, $\alpha = 1.25$, and $\alpha = 0.8$ at 1000 r/min. It is shown that dead-beat controller is not able to track its reference accurately if model mismatch occurs. Compared to the proposed current controller, the equivalent gain of dead-beat controller is $k = 10000$, which is much larger than the gain used in the proposed controller. Larger gain indicates that the dead-beat controller is more sensitive to noise.

Then, the SRM is controlled to run at mechanical speed of 6000 r/min. It has to be noted that when the mechanical speed is 6000 r/min, the electric speed of this SRM is 48 000 r/min. Normally, single pulse operation have to be applied at such high speed, while the proposed controller still shows its effectiveness. Fig. 14 shows the phase current, current reference, the original flux linkage reference $\psi_m(\theta, i_{ref})$, and the adjusted flux linkage reference $\psi_{adj}(\theta, i_{ref})$. In comparison, Fig. 15 shows the phase current and its reference with hysteresis controller.
It is shown from Figs. 14(a) and 15 that the proposed current controller also has fast dynamic response as hysteresis controller at higher speed. But at high speed, as shown in Fig. 15, the reference current profile for hysteresis controller have to be modified so that the turn on angle is advanced. Otherwise the phase current can not reach its reference. However, due to Fig. 6, the reference flux is calculated in advance in the proposed controller. The phase winding is turned on automatically. Due to the limit of dc link voltage, the current can not reach its reference somewhere at high speed, but the controller is still effective. Fig. 14(b) shows that when the calculated \( u_c \) exceeds \( U_{DC} \), the flux linkage reference is adjusted according to Fig. 6 to keep the parameter adaption functioning.

**B. Experimental Results**

Experiments are designed to verify the effectiveness of the proposed current method. Fig. 17 shows the diagram of the experimental setup. The studied SRM is controlled by a control board with a DSP on it. The controller takes the reference torque \( T_{ref} \), feed it into a linear torque distributor to get the reference current for each phase. The proposed current controller takes the reference current and generate switching signals to an asymmetric half bridge. The shaft of the SRM is connected with a brushless dc (BLDC) machine. The BLDC is connected with a passive rectifier. The output of the rectifier is connected with a dc/dc converter. The dc/dc converter is connected with a load resistor. In the experiment, \( T_{ref} = 3.2 \text{ N} \cdot \text{m} \) is given into the controller. Load torque on the SRM is adjusted by adjusting the duty ratio of the dc/dc converter. The duty ratio is given by a variable resistor connected with the dc/dc converter. In this way, the speed of the SRM could be adjusted. Fig. 18(a) shows the studied SRM and the BLDC load. Fig. 18(b) shows the control board, asymmetric half bridge for SRM, the dc/dc converter, and the load resistor. The flux linkage profile of the studied SRM is shown in Fig. 2. Parameters of this experiment are the same as in the simulation, as shown in Tables II and III.

The SRM control algorithm, including the torque sharing and the proposed current controller, is implemented in TI’s DSP TMS320F28335. The frequency of the DSP is 150 MHz. The longest possible execution time of the control algorithm with proposed current controller is shown in Fig. 16. It takes around 5 \( \mu \text{s} \) to prepare the data of three phase current, rotor position, and rotor speed. Then, at \( t(k-1) \), it takes about 4.28 \( \mu \text{s} \) to calculate \( \psi_m(\theta(k-1),i(k-1)) \) for three phases. At \( t(k-1/2) \), it takes 17.25 \( \mu \text{s} \) to complete the control algorithm without any code optimization. If the algorithm code is further optimized, it is possible to implement the proposed controller in popular low cost microcontrollers in market.

First, duty ratio of the dc/dc converter is adjusted to control the SRM to run at mechanical speed of 1000 r/min. The proposed current controller and hysteresis current controller are applied, respectively. The parameters of the two controllers are the same as they are in the simulation. Fig. 19 shows the waveforms of the two current controllers. It is shown that the waveforms obtained in experiments matches the simulation results very much. The proposed current controller has almost the same dynamic response and accuracy as that of the hysteresis controller. However, the proposed controller has lower current ripple and needs...
Fig. 17. Diagram of the experimental setup.

Fig. 18. Experimental setup. (a) Studied SRM and its load. (b) Digital controller and dc/dc converter.

Fig. 19. Experimental phase current (CH1), voltage (CH2) and current reference (marked by red line) waveforms at 1000 r/min. (a) Waveforms of proposed current controller. (b) Waveforms of hysteresis current controller.

Fig. 20. Experimental three-phase current (CH1, CH2, CH3), torque reference (marked by red line, 2 N-m/div), and total torque production (marked by black line, 2 N-m/div) waveforms at 1000 r/min. (a) Waveforms of proposed current controller. (b) Waveforms of hysteresis current controller.
less sampling rate. Fig. 20 shows three-phase current waveforms of the two controllers. Total torque productions by the two controllers are calculated from their corresponding current waveforms. It is shown that the reduced current ripple by the proposed current controller also results in reduced torque ripple.

Then, the SRM is controlled to run at mechanical speed of 6000 r/min. The waveforms of the proposed current controller and hysteresis current controller are shown in Fig. 21. It is shown that the proposed current controller works well even at high speed. The dynamic performance of the proposed controller is also the same as of the hysteretic controller with modified reference current shown in Fig. 10.

The estimated parameters at 1000 r/min are \( \alpha = 0.980, \beta = 0.640, \gamma = 1.02 \). When the SRM is running at 6000 r/min, the final estimated parameters are \( \alpha = 0.969, \beta = 0.635, \gamma = 1.11 \). The estimated parameters while running at low speed and high speed are very close. Also they are close to the values obtained in simulation as shown in Fig. 7. This also verifies the effectiveness of the proposed controller.

VI. CONCLUSION

In this paper, a PWM current controller for SRM drives is proposed to replace the conventional hysteretic current controller. With parameter adaption, both fast dynamics and stability are guaranteed. Relationship between the proposed controller and previously proposed methods is discussed. An improved sampling method is proposed to avoid PWM delay in the control loop. The proposed controller is digitally implemented and keeps the similar dynamics and accuracy as digital hysteretic controller. However, the proposed current controller shows its advantages of lower current ripple and lower sampling rate over hysteretic controller in both the simulations and experiments under various testing conditions.

REFERENCES


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